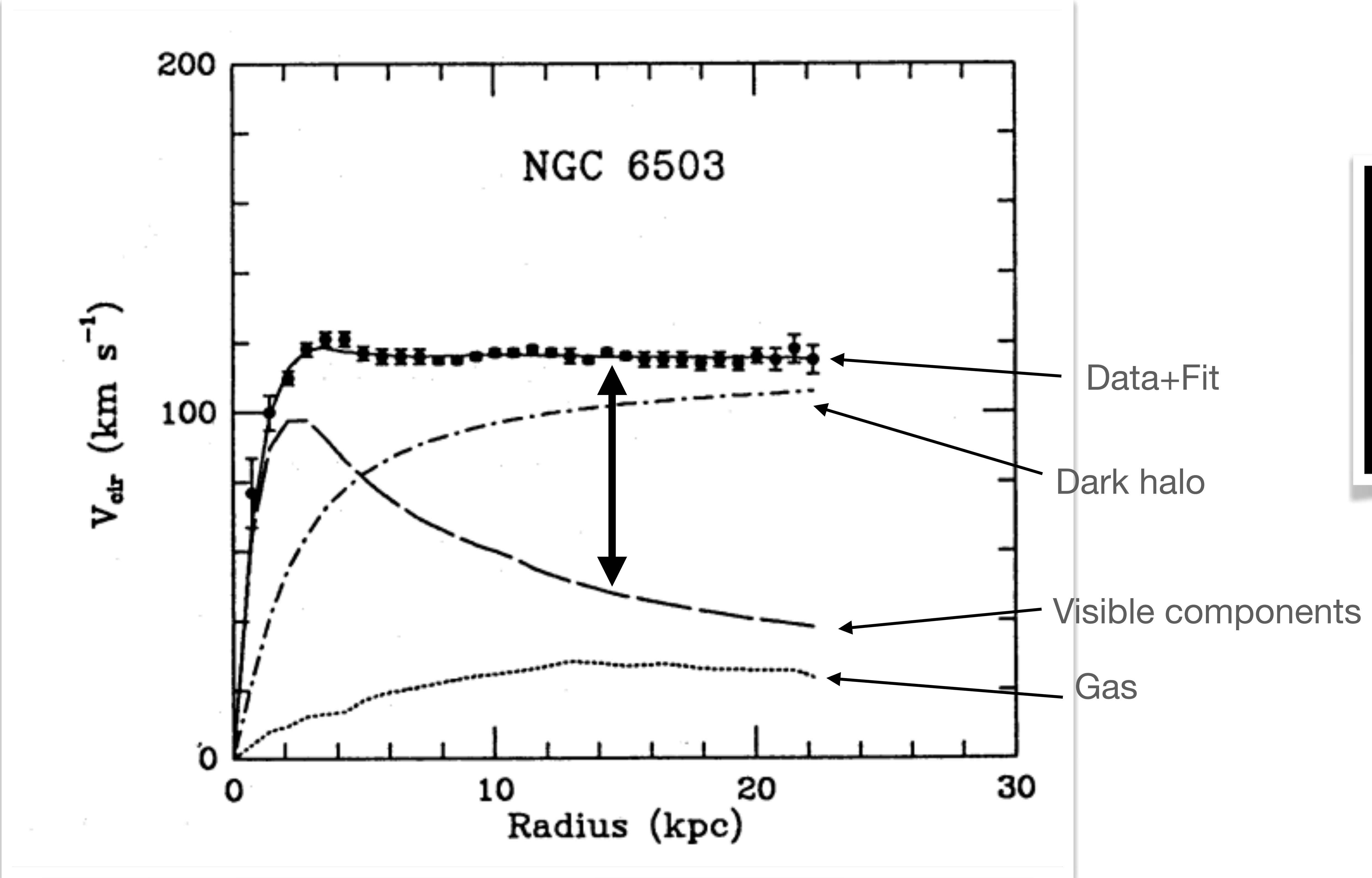

QSFP Workshop

Axions and Wave-like Dark Matter Tutorial

Chelsea Bartram

September 7 2021

Overview

- Basic understanding of how axion haloscopes work.
- Equivalent circuit for the axion haloscope.
- Computing the sensitivity of the axion haloscope.
- Computing the doppler shift of the axion signal.
- Overview of coupled oscillators.

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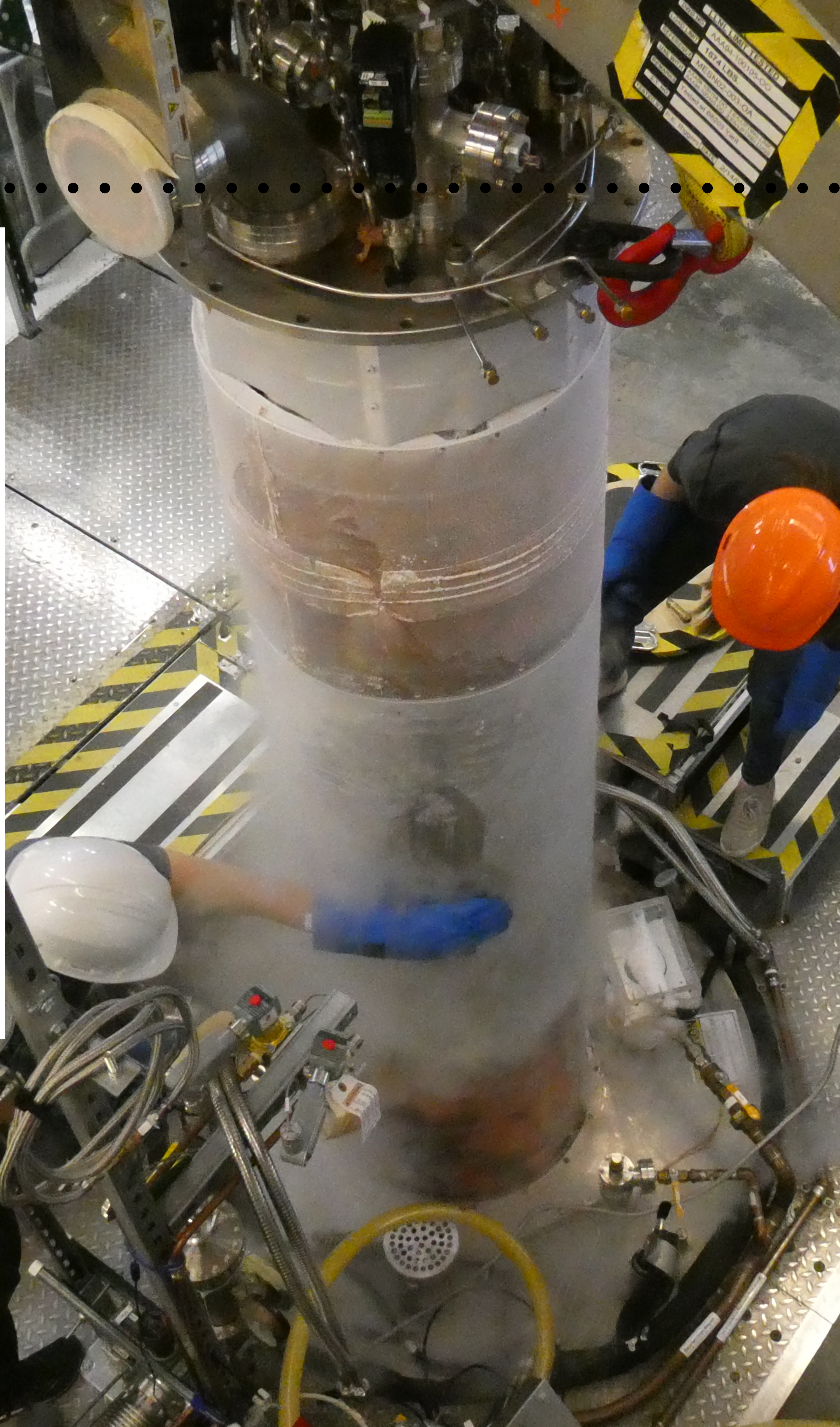

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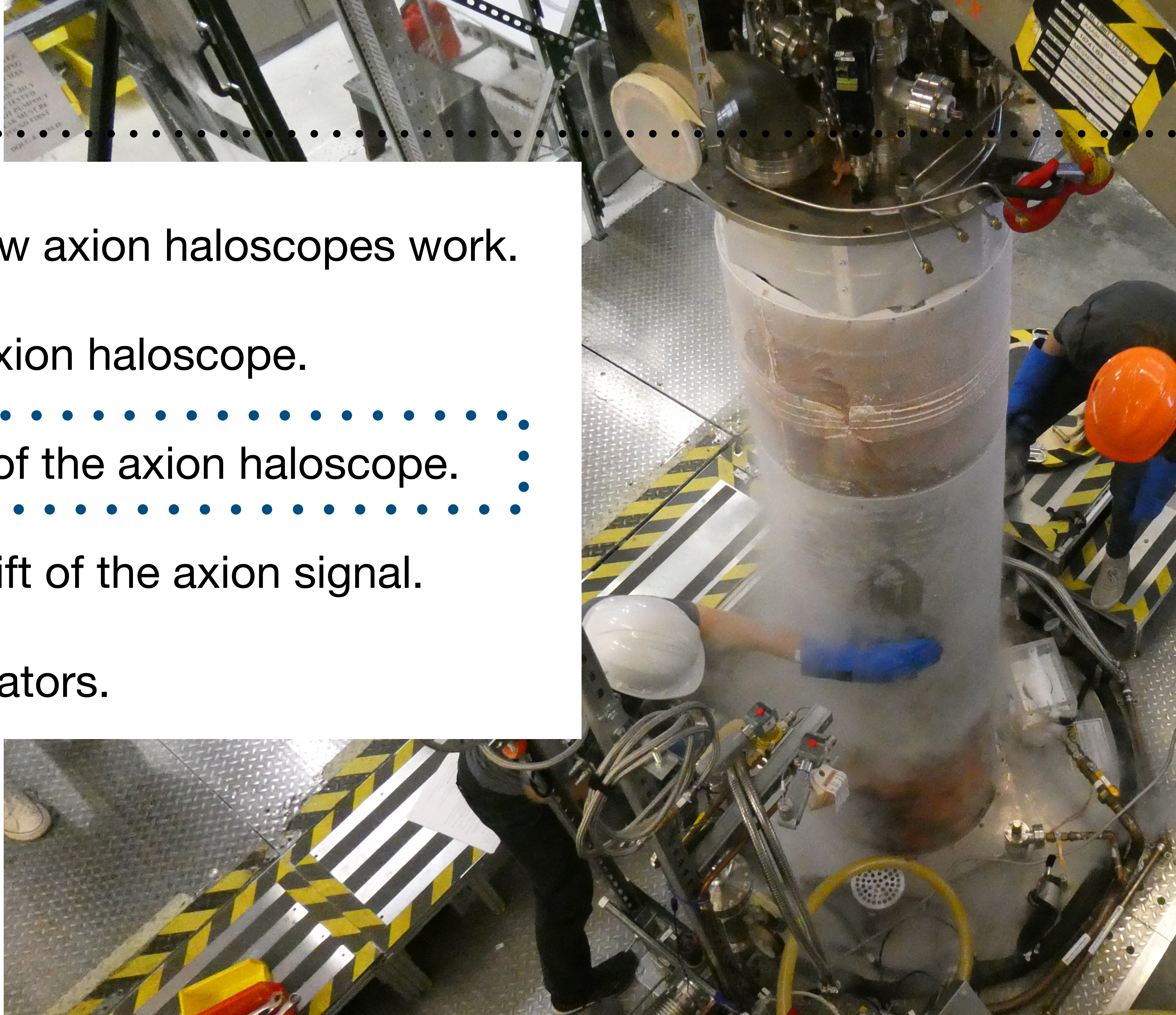

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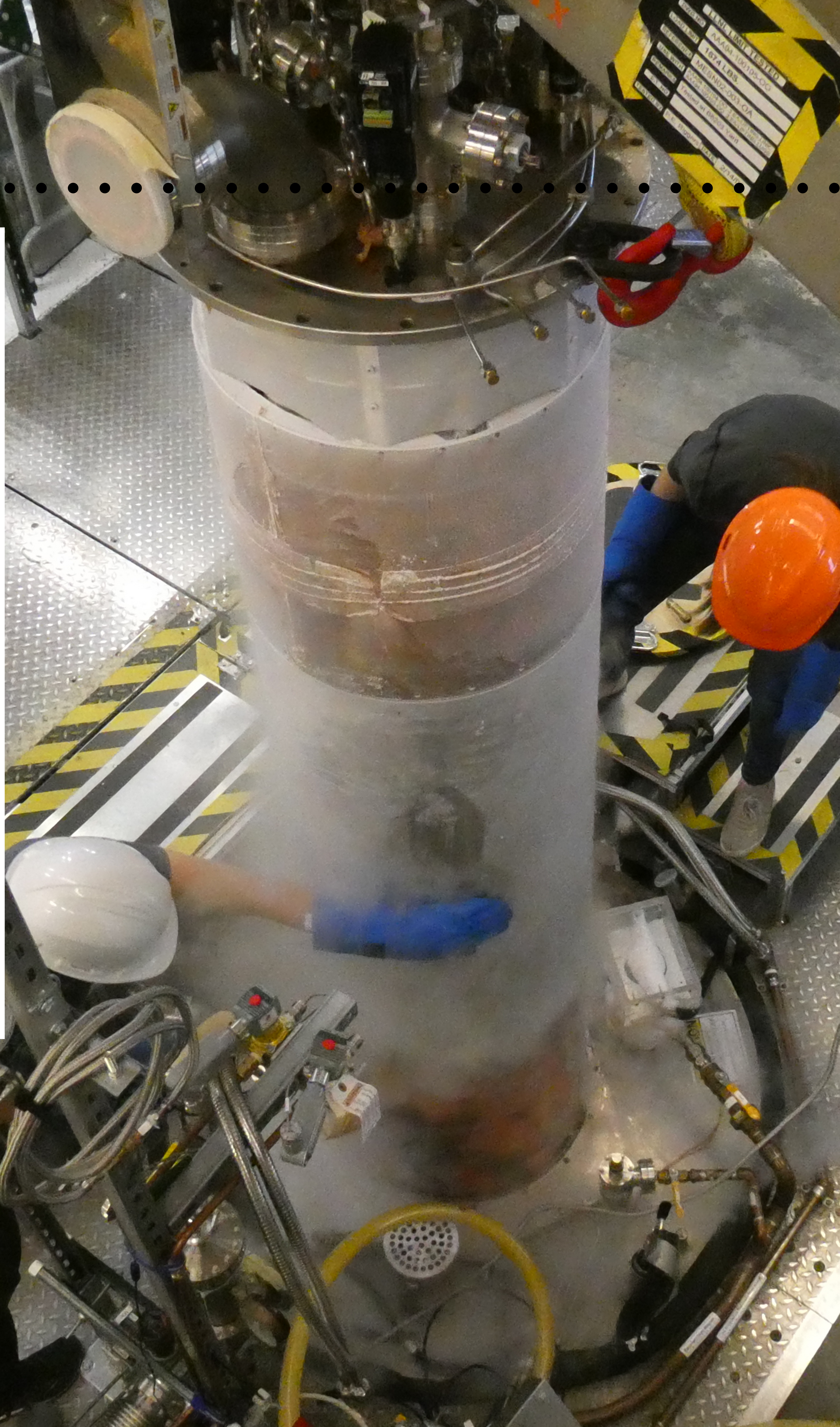

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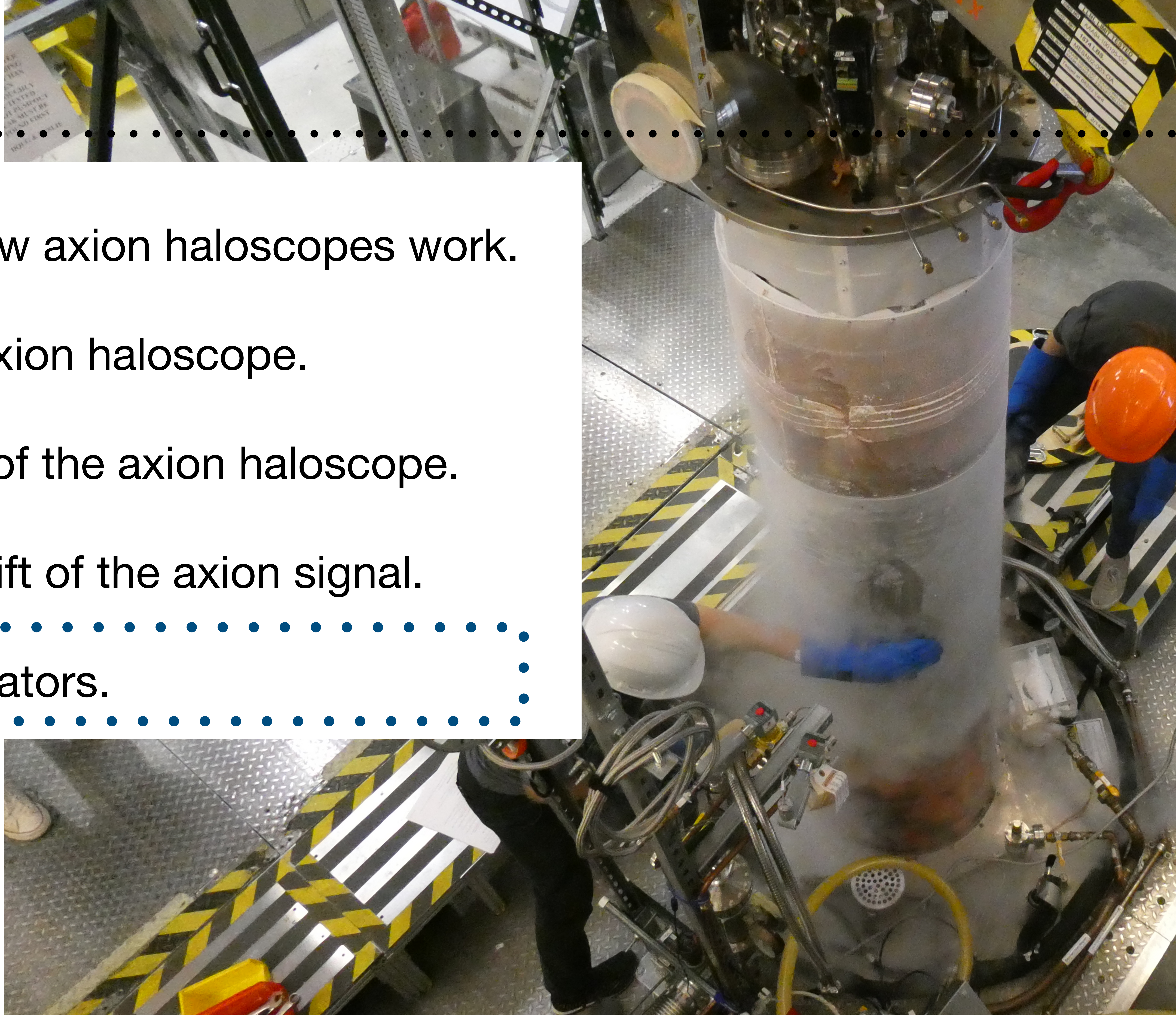
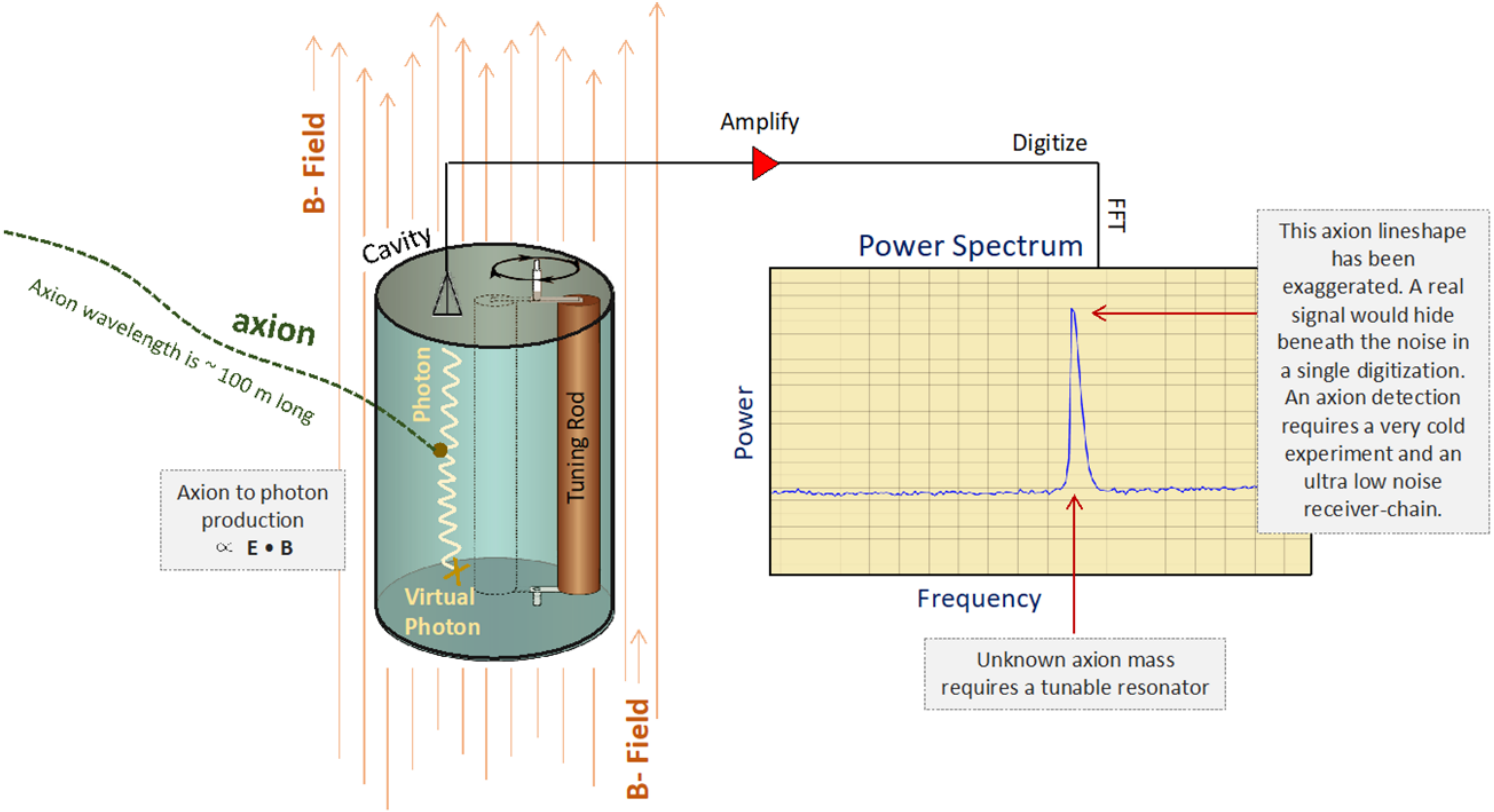
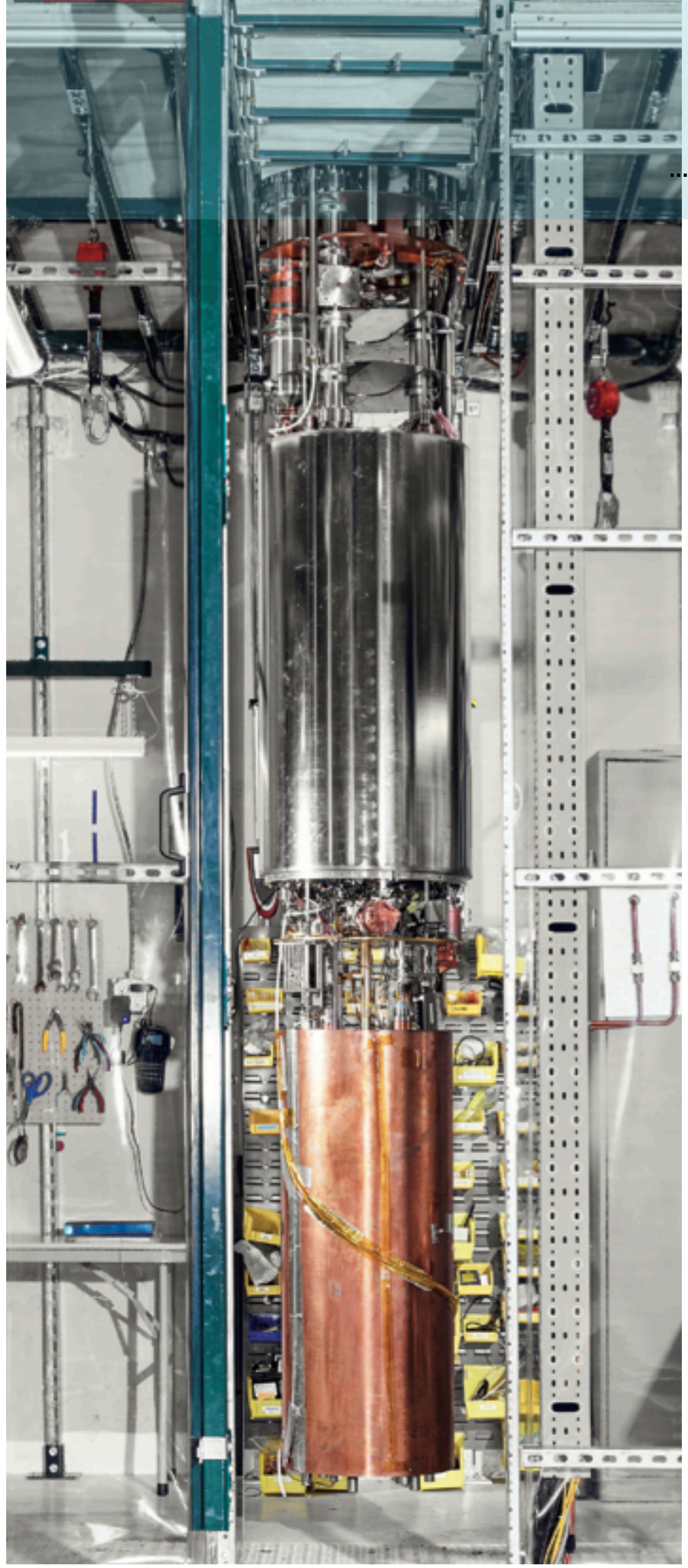


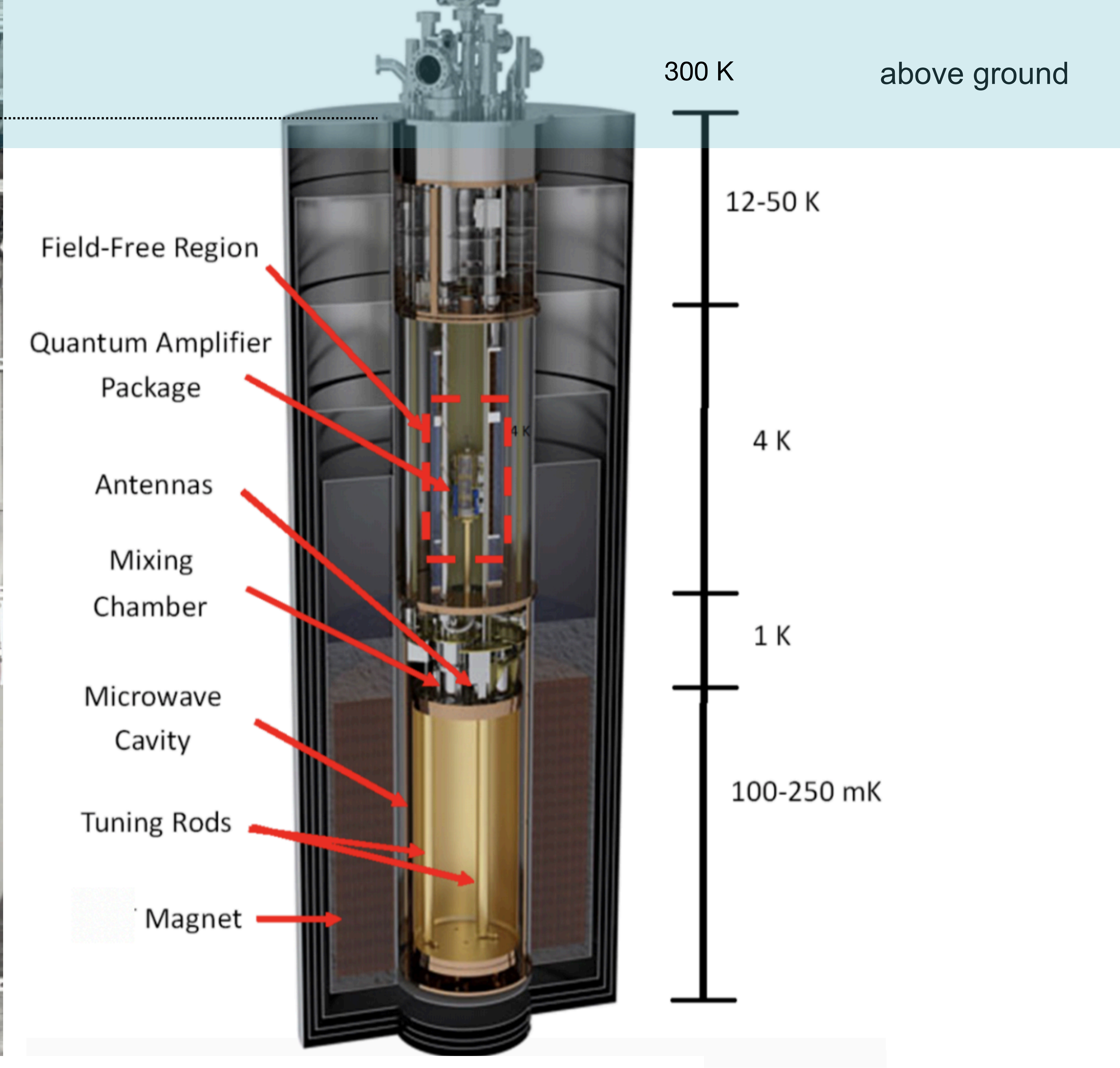
The Axion Haloscope



ADMX

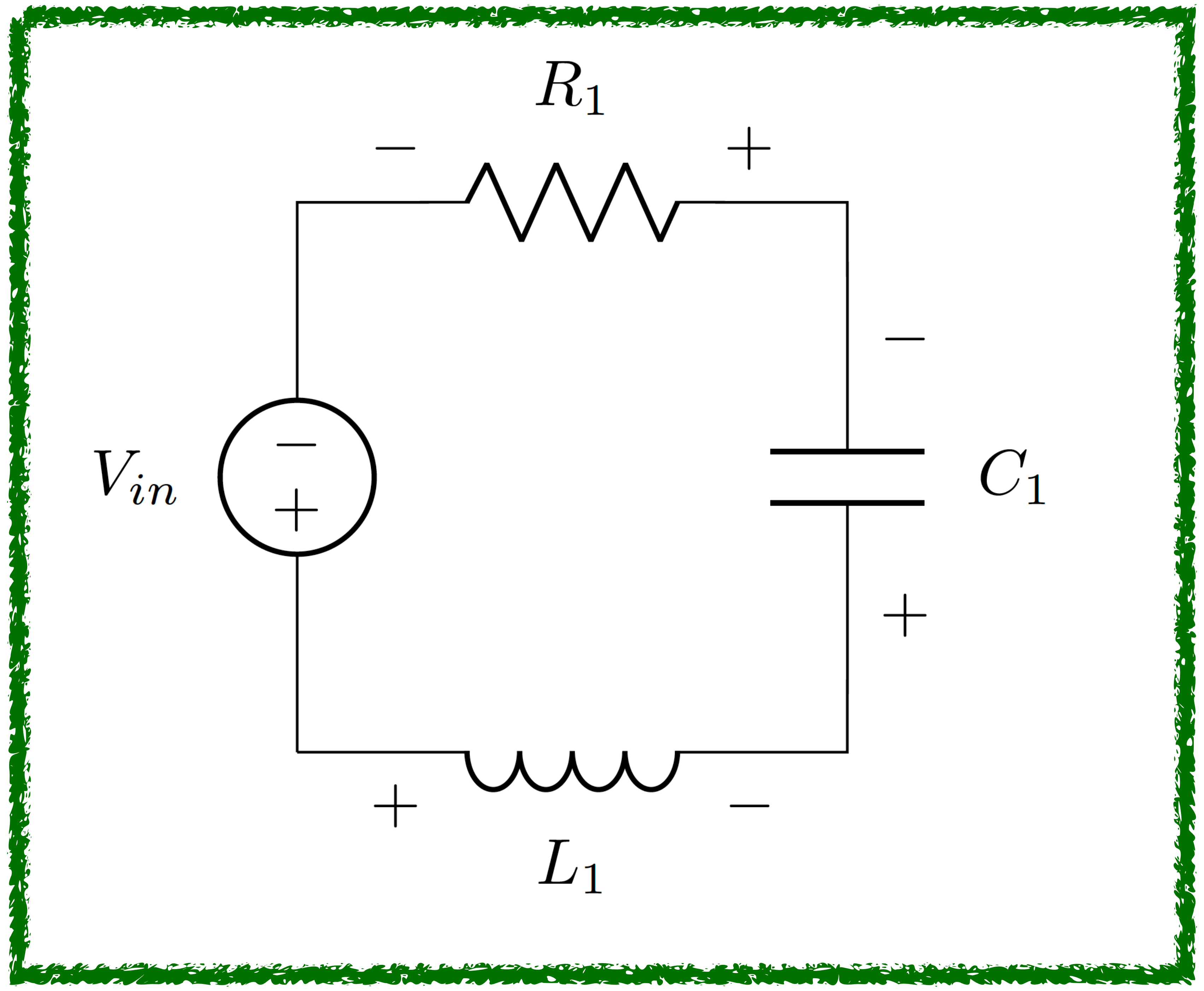
- Dil Fridge: Reaches ~100 mK
- Superconducting magnet:
  ~can reach up to 8 T
- Quantum electronics: Josephson Parametric Amplifier (JPA)
- Field cancellation coil
- Microwave cavity and electronics





Equivalent circuit model for axion detectors

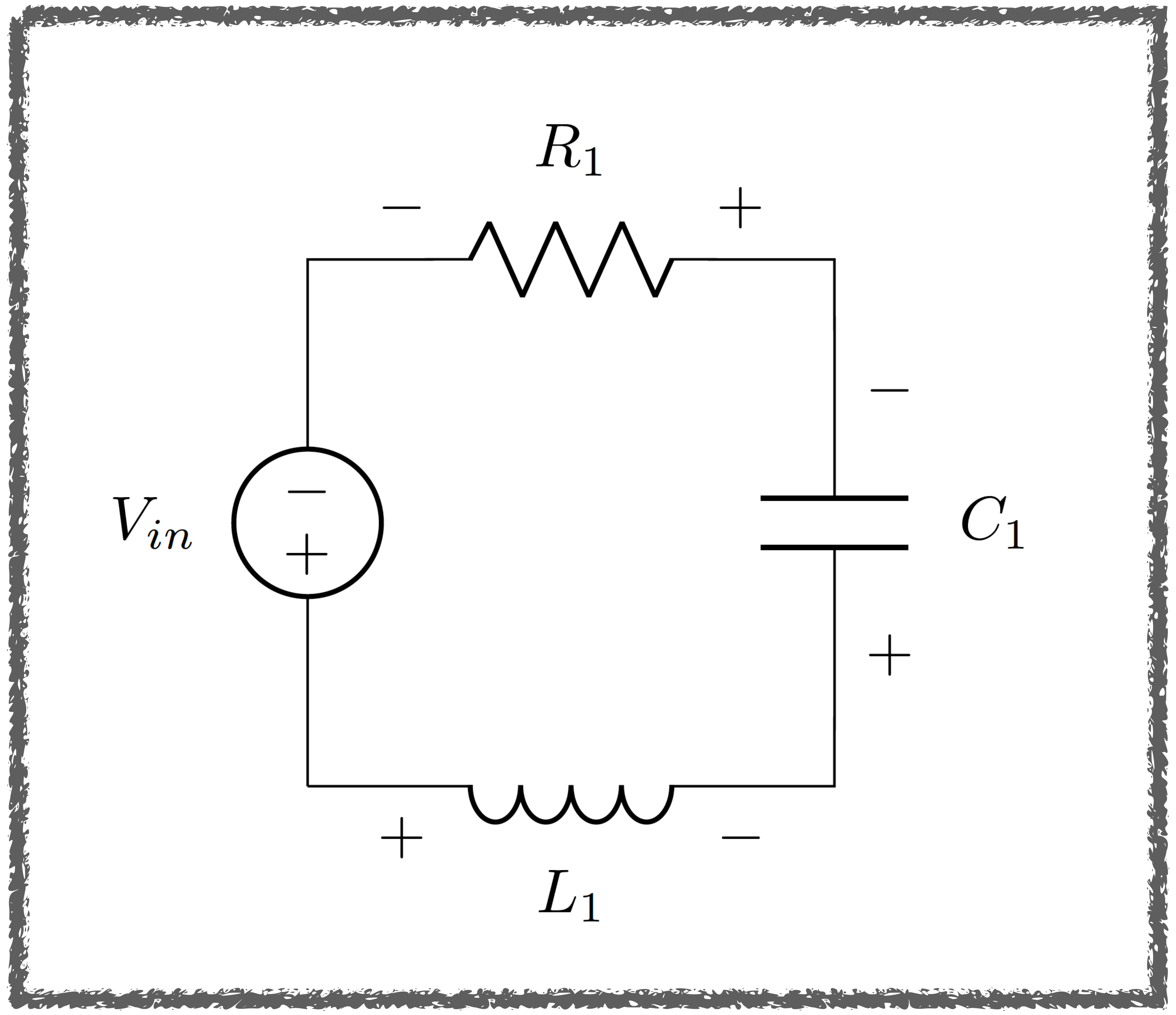
- When looked at from any one pair of terminals, any arbitrary circuit made of ideal impedances and generators is, at any given frequency, equivalent to a generator epsilon in series with an impedance z.

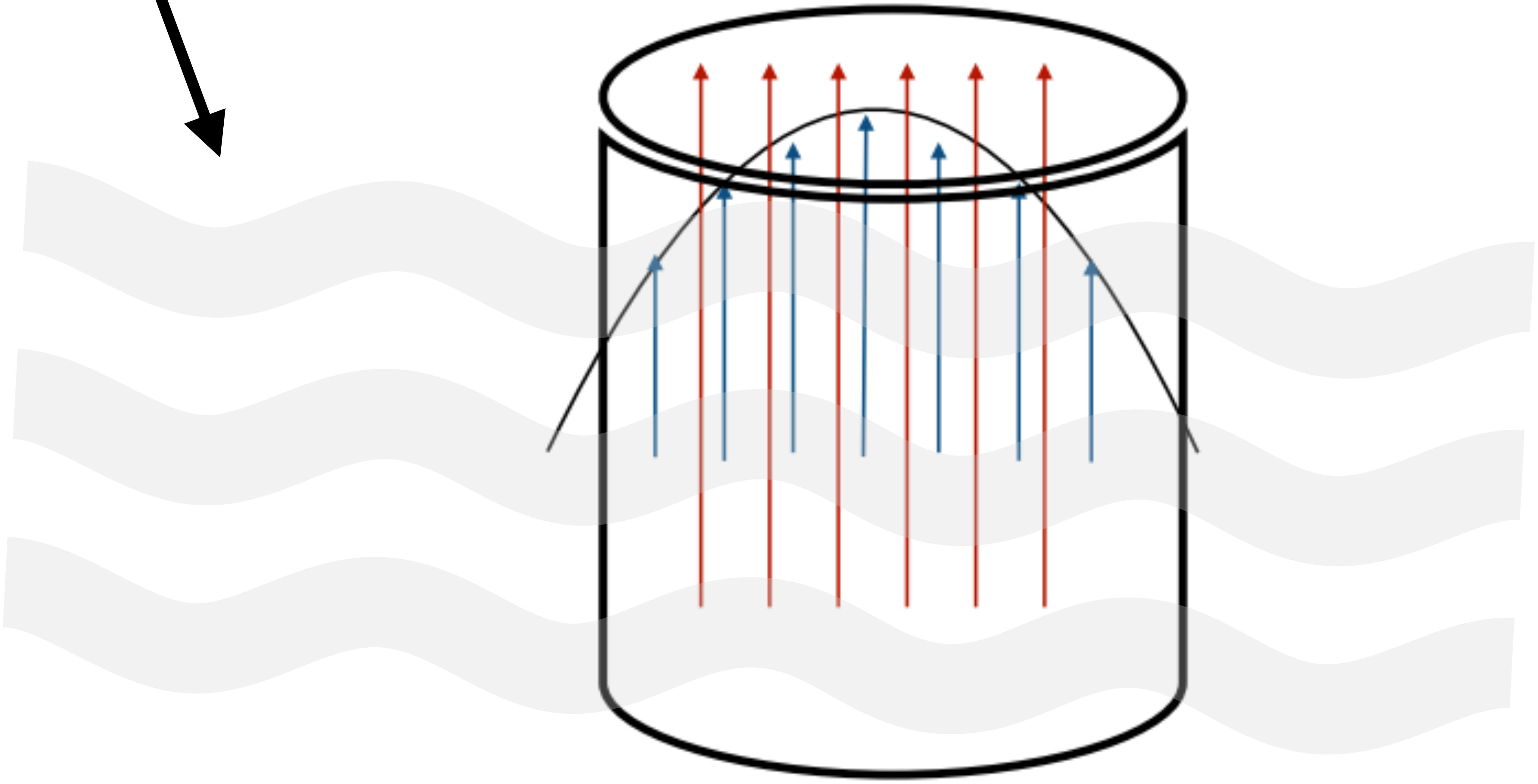


Equivalent circuit of a cavity is an RLC circuit!

Equivalent circuit model for axion detectors

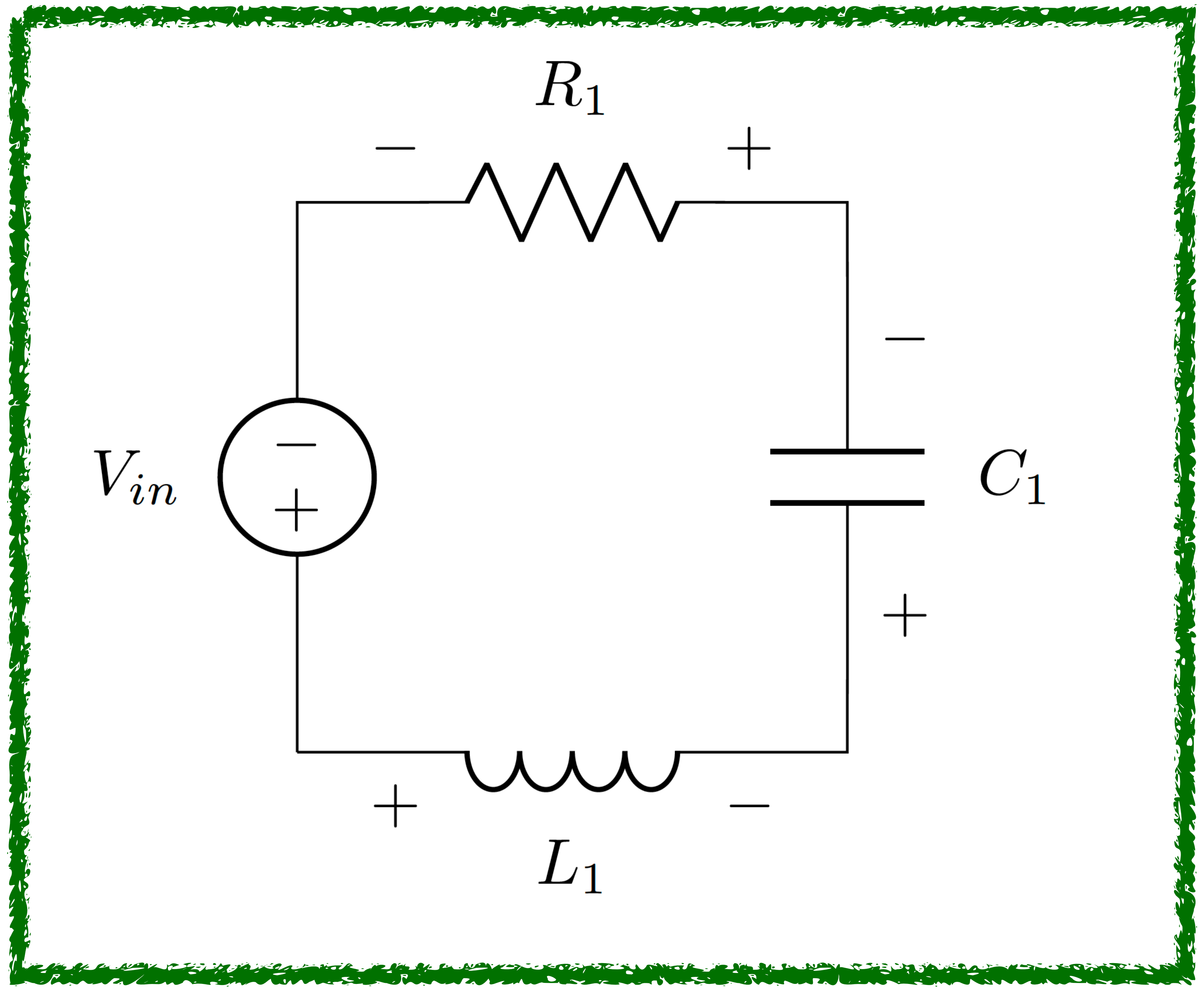
In a cavity axion detector the source is the axion field coupling to modes in the resonant cavity.





Red is cartoon magnetic field
Blue is cartoon electric field

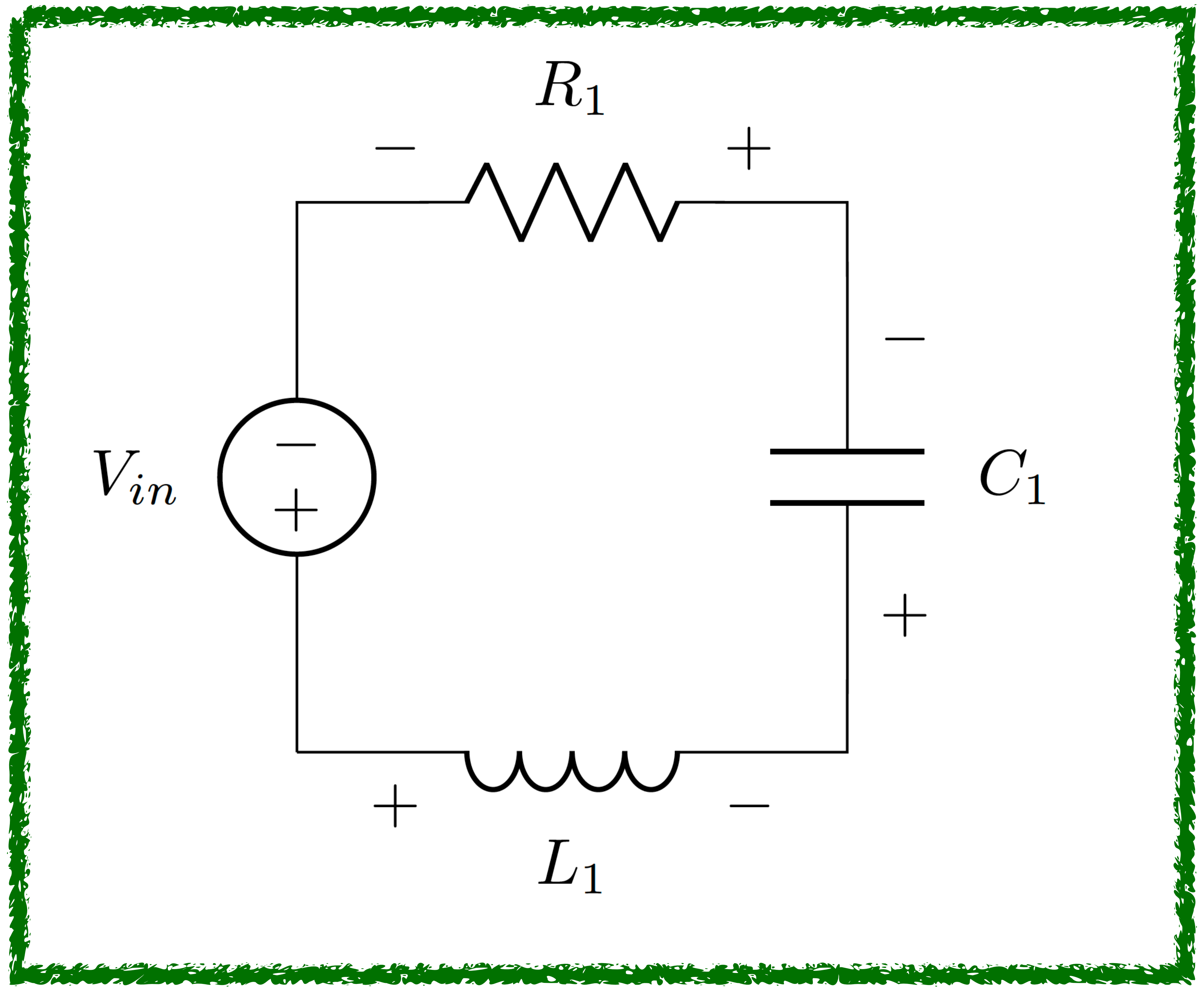
Equivalent circuit model for axion detectors



$$L = \frac{L}{2}\dot{q}^2 - \frac{1}{2C}q^2 + qV(t)$$

Write out the Lagrangian for the circuit

Equivalent circuit model for axion detectors

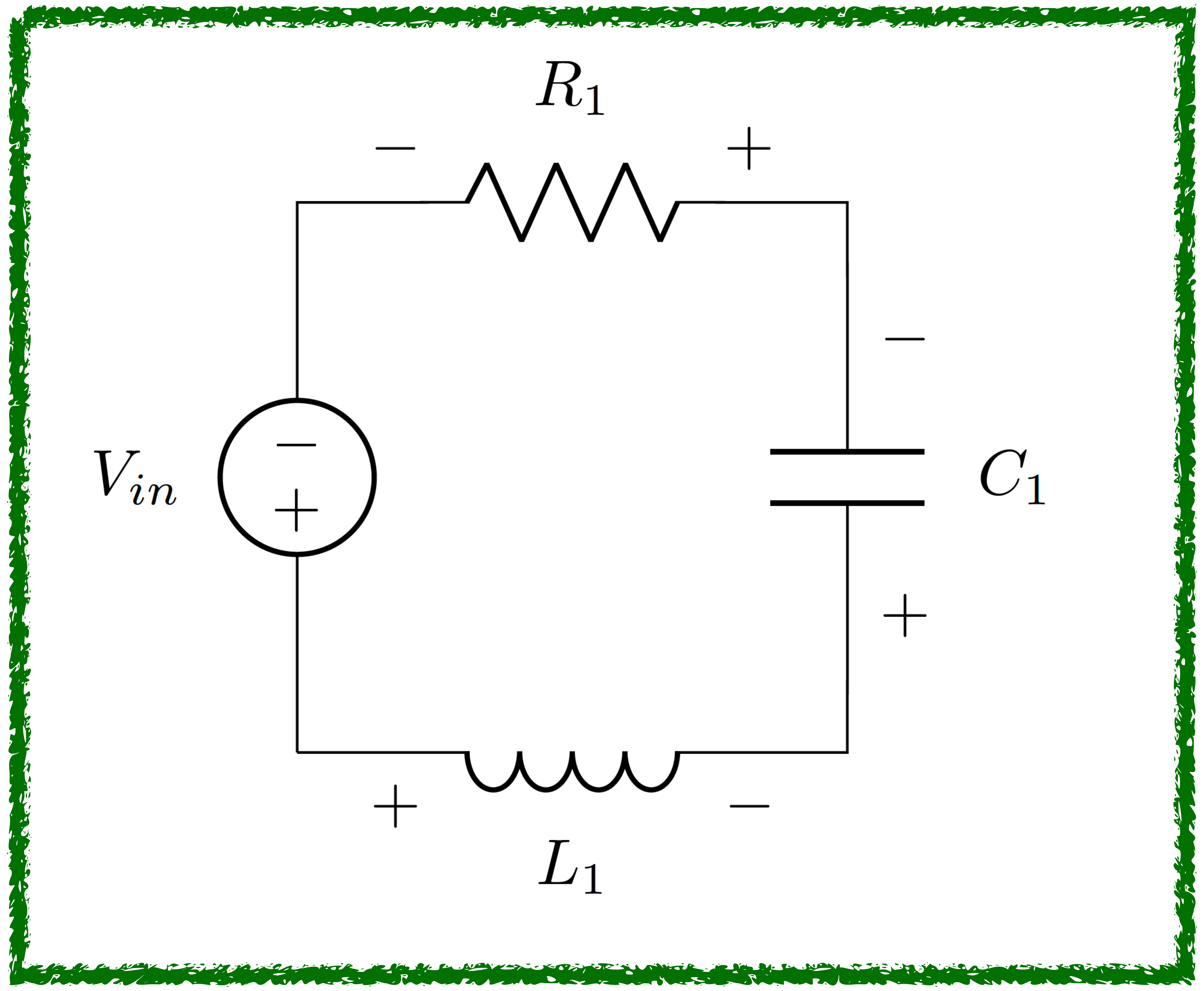


$$L = \frac{L}{2}\dot{q}^2 - \frac{1}{2C}q^2 + qV(t)$$

$$F = \frac{1}{2}R\dot{q}^2$$

Write down the dissipative term from the resistor

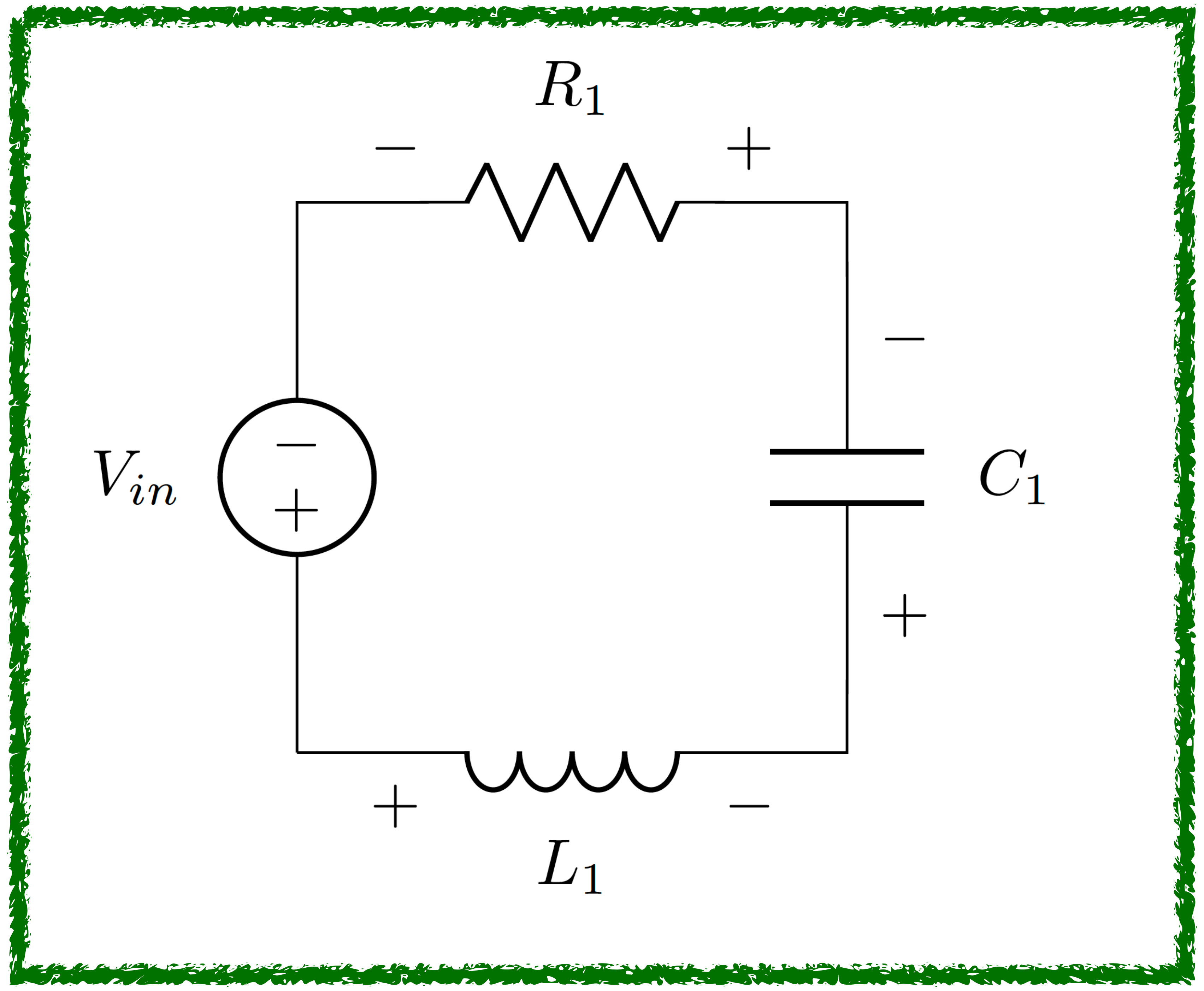
Equivalent circuit model for axion detectors



$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial F}{\partial \dot{q}} = 0$$

Write down the Euler-Lagrange Equation

Equivalent circuit model for axion detectors

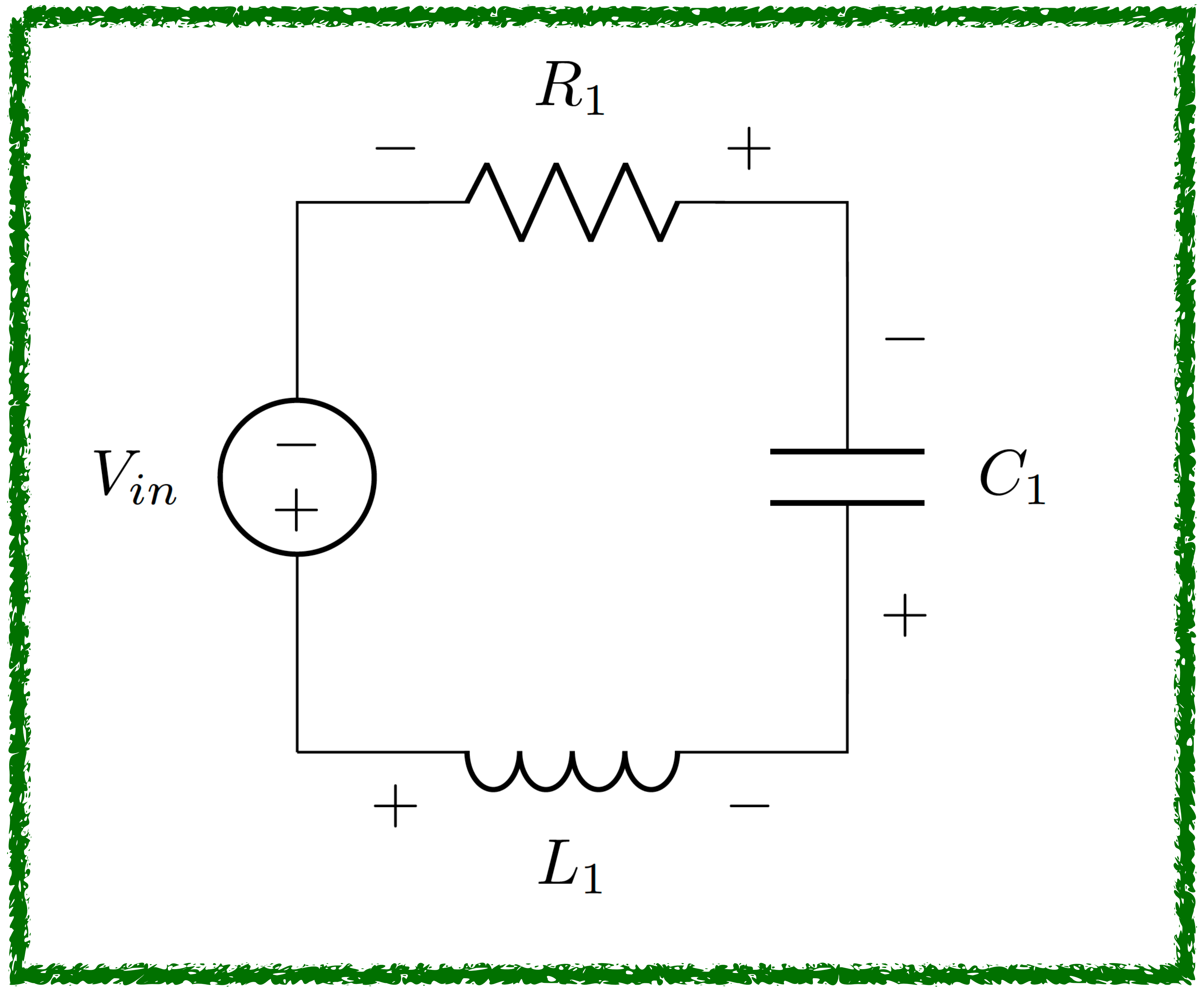


$$L\ddot{q} - \frac{q}{C} + V(t) + R\dot{q} = 0$$

And compute it...

Equivalent circuit model for axion detectors

- The Lagrangian describing the coupling of the axion to the electric and magnetic fields is as follows:



$$\mathscr{L} = -\epsilon_0 g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

Equivalent circuit model for axion detectors

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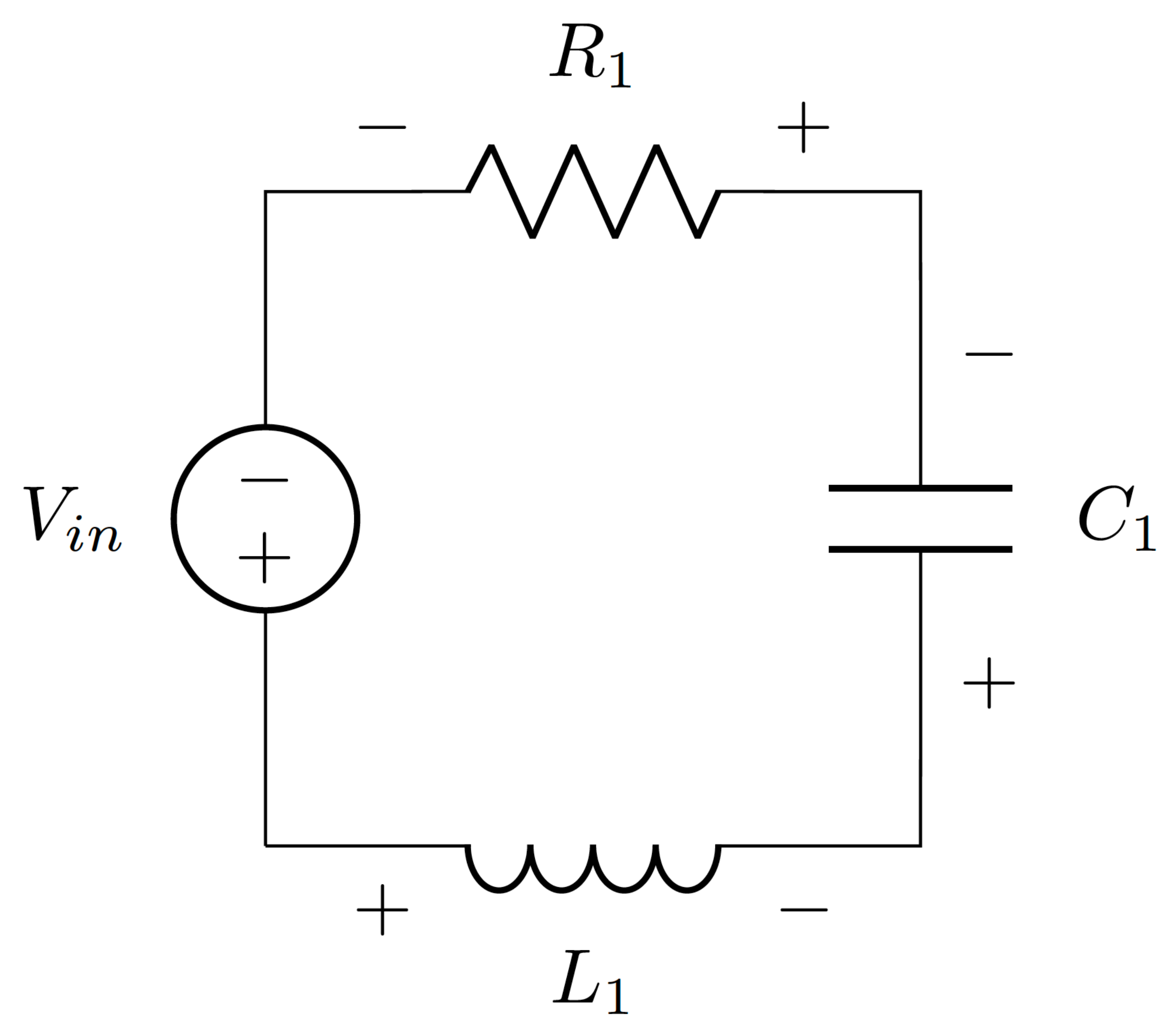


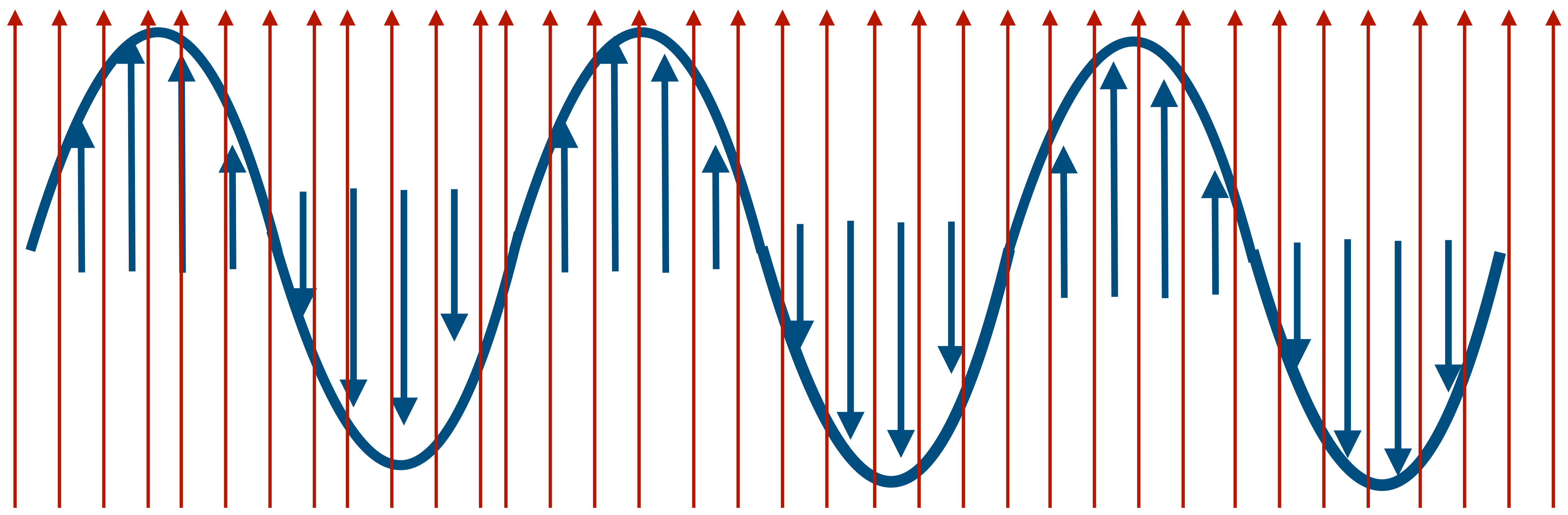
$$\mathscr{L} = -\epsilon_0 g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

$$q(t)V(t) = -g_{a\gamma\gamma} a(t) c \epsilon_0 \int_V dV \vec{E}(\vec{x}, t) \cdot \vec{B}(\vec{x}, t)$$

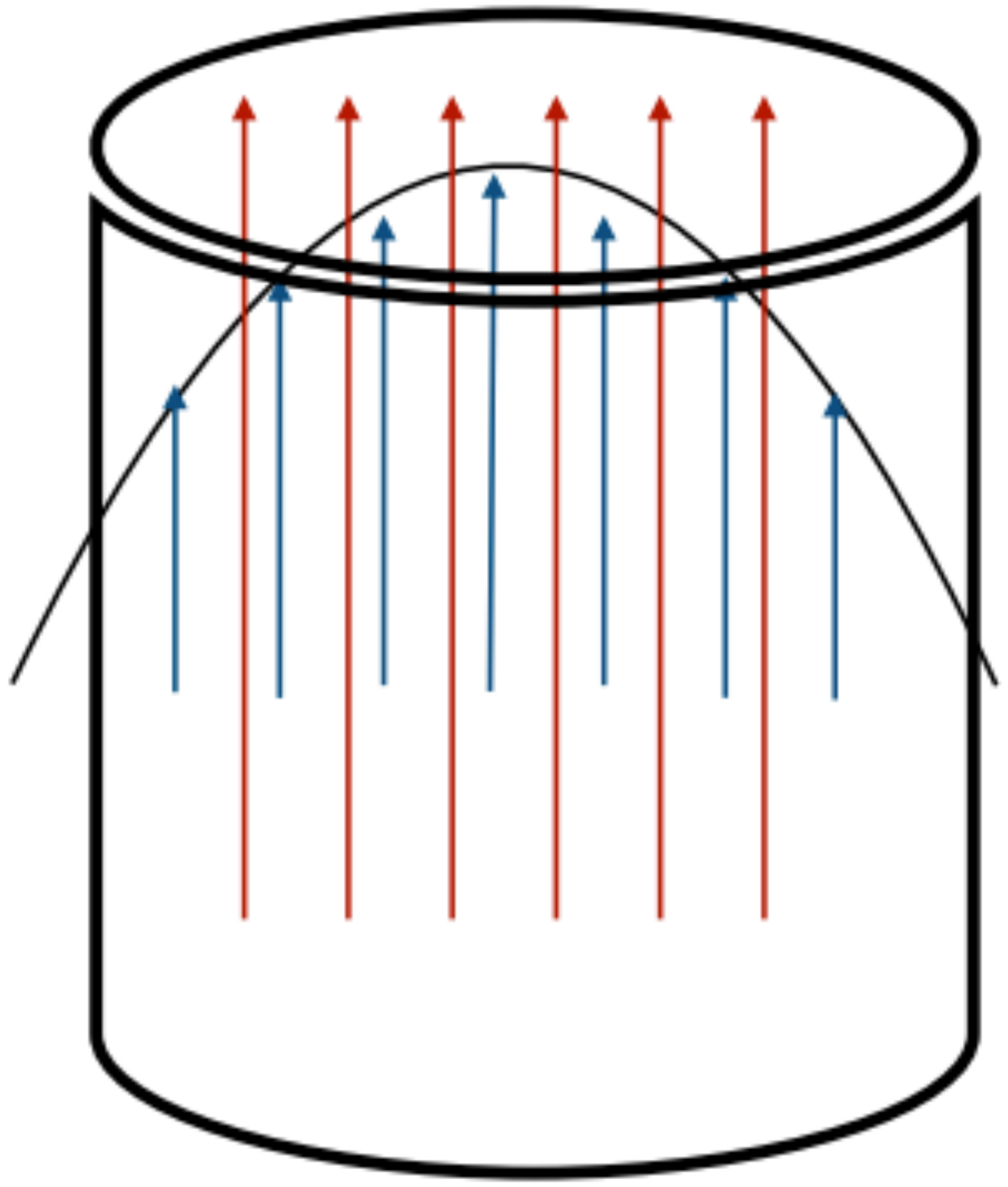
Integral of this over the cavity volume is equal to the source in the equivalent circuit

Equivalent circuit model for axion detectors

- Compute the form factor.
- Form factor is the overlap of the electric and magnetic fields.
- Can you think of different ways to manipulate the form factor?



$$f_{nlm} = \frac{\left(\int_V \vec{E}(\vec{x},t)\cdot\vec{z}\right)^2}{V\int_V dV\,\epsilon_r E^2}$$



Red is cartoon magnetic field
Blue is cartoon electric field

Equivalent circuit model for axion detectors

$$f_{nlm} = \frac{\left(\int_V \vec{E}(\vec{x},t)\cdot\vec{z}\right)^2}{V\int_V dV\,\epsilon_r E^2}$$

$$\frac{q^2}{2C} = \frac{\epsilon_0}{2}\int_V dV\,\epsilon_r E^2$$

Form factor

Charge - Energy density relationship

$$\int_V dV\,\vec{E}(\vec{x},t)\cdot\vec{z} = q\sqrt{\frac{f_{nlm}V}{\epsilon_0 C}}$$

Equivalent circuit model for axion detectors

Substitute that equation into the equation for the source term

$$\int_V dV \vec{E}(\vec{x},t)\cdot\vec{z} = q\sqrt{\frac{f_{nlm}V}{\epsilon_0 C}}$$

$$q(t)V(t) = -g_{a\gamma\gamma}a(t)c\epsilon_0 B_0\int_V dV \vec{E}(\vec{x},t)\cdot\vec{z}$$

$$V(t) = -g_{a\gamma\gamma}cB_0\sqrt{\frac{f_{nlm}V\epsilon_0}{C}}a(t)$$

Equivalent circuit model for axion detectors

$$V(t) = -g_{a\gamma\gamma} c B_0 \sqrt{\frac{f_{nlm} V \epsilon_0}{C}} a(t)$$

We know that Power = V^2/R, so let's compute the power!

$$P = \frac{\langle V^2(t) \rangle}{R} = g_{a\gamma\gamma}^2 c^2 \epsilon_0 B_0^2 V f_{nlm} \frac{1}{RC} \langle a^2(t) \rangle$$

Equivalent circuit model for axion detectors

$$P = \frac{\langle V^2(t)\rangle}{R} = g_{a\gamma\gamma}^2 c^2 \epsilon_0 B_0^2 V f_{nlm} \frac{1}{RC} \langle a^2(t)\rangle$$

Substitute in the definition of quality factor and resonant frequency for an RLC circuit

High Q —> Higher Probability of Photon/Axion Bounces —> Higher Conversion Rate

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \frac{1}{RC} = \omega_0 Q$$

$$P = g_{a\gamma\gamma}^2 c^2 \epsilon_0 B_0^2 V f_{nlm} \omega_o Q \langle a^2(t)\rangle$$

Equivalent circuit model for axion detectors

$$P = g_{a\gamma\gamma}^2 c^2 \epsilon_0 B_0^2 V f_{nlm} \omega_o Q \left\langle a^2(t) \right\rangle$$

$$\left\langle a^2(t) \right\rangle = \frac{\rho_a \hbar^2}{m_a^2 c}$$

$$P = g_{a\gamma\gamma}^2 c^2 \epsilon_0 B_0^2 V f_{nlm} \omega_o Q \; \frac{\rho_a \hbar^2}{m_a^2 c}$$

Equivalent circuit model for axion detectors

$$P = g_{a\gamma\gamma}^2 c^2 \epsilon_0 B_0^2 V f_{nlm} \omega_o Q \frac{\rho_a \hbar^2}{m_a^2 c}$$

$$g_{a\gamma\gamma}(\mathrm{GeV}^{-1}) = 10^{-7}\mathrm{GeV}^{-1}\left(\frac{M_a}{0.62\,\mathrm{eV}}\right)\frac{\alpha g_\gamma}{\pi}$$

$$\frac{g_{a\gamma\gamma}^2 \rho_a}{m_a^2} = (3.64\times10^{-19})^2\mathrm{eV}^{-4}(3.44\times10^{-6})\mathrm{eV}^4 = 4.56\times10^{-43}$$

$$P = (2.86\times10^{-42})c^2\epsilon_0 B_0^2 V f_{nlm} f_a Q$$

Power in terms of physical parameters of the receiver

$$P_{a\to\gamma} = 1.52\times10^{-21}\mathrm{W}\left(\frac{V}{220\ell}\right)\left(\frac{B}{7.6\mathrm{T}}\right)^2 f_{nlm}\left(\frac{g_\gamma}{0.97}\right)^2\left(\frac{\rho_a}{0.45\,\mathrm{GeV/cc}}\right)\left(\frac{f_a}{750\,\mathrm{MHz}}\right)\left(\frac{Q}{70,000}\right)$$

Cavity Volume

Magnetic Field

Form Factor

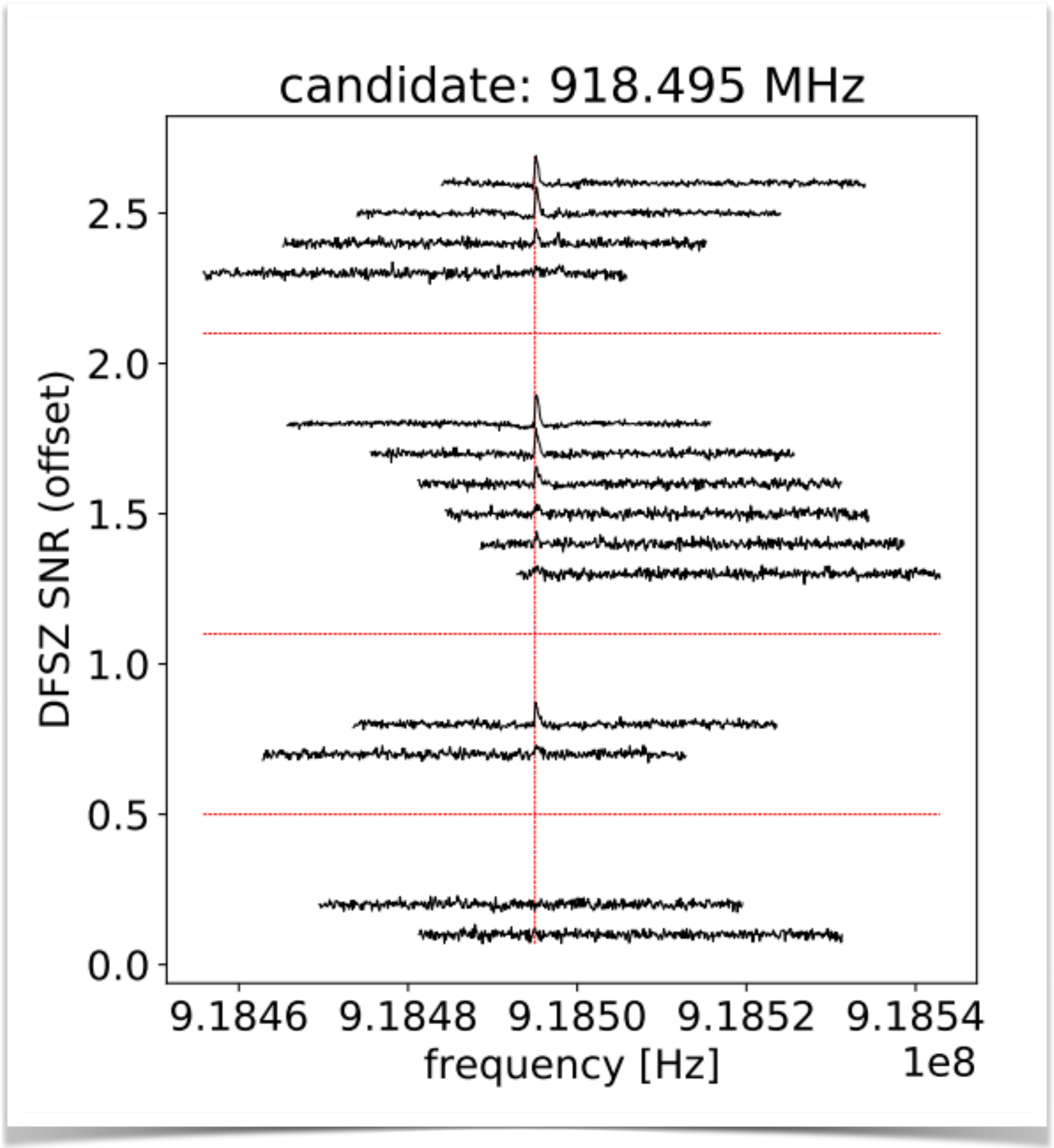
Model Dependent Term

Dark Matter Density

Axion Frequency

Quality

Signal-to-noise



Example synthetic candidate spectra

- Johnson noise = thermal noise associated with the resistance of the cavity walls
- Assume total noise may be modeled as Johnson noise at some effective temperature Tsys
- Aka, the amplitude distribution of the noise voltage within a bandwidth b is Gaussian
- Noise power is proportional to the variance of this voltage distribution
- Standard error of the variance of a Gaussian gives us:

$$\delta P_N = \sqrt{\frac{2}{n-1}} k_B T_{\mathrm{sys}} \Delta\nu \qquad n = 2\Delta\nu\tau$$

$$\delta P_N = \frac{k_B T_{\mathrm{sys}} \Delta\nu}{\sqrt{\Delta\nu\tau}}$$

N is the number of independent samples drawn from the Gaussian distribution

Computing the scan speed and sensitivity

$$\frac{df}{dt} \approx 543\,\frac{\mathrm{MHz}}{\mathrm{yr}} \left(\frac{B}{7.6\,\mathrm{T}}\right)^4 \left(\frac{V}{136\,\ell}\right)^2 \left(\frac{Q_l}{30000}\right) \left(\frac{C}{0.4}\right) \left(\frac{g_\gamma}{0.36}\right)^4 \left(\frac{f}{740\,\mathrm{MHz}}\right)^2 \left(\frac{\rho}{0.45\,\mathrm{GeV/cm^3}}\right)^2 \left(\frac{0.2\,\mathrm{K}}{\mathrm{T_{sys}}}\right)^2 \left(\frac{3.5}{\mathrm{SNR}}\right)^2$$

Maximize	Can't Control	Minimize
• B Field	• Frequency	• System noise:
• Volume	• Coupling	• Amplifier Noise
• Quality Factor	• Dark Matter Density	• Physical Noise
• Form Factor		

Computing the scan speed and sensitivity

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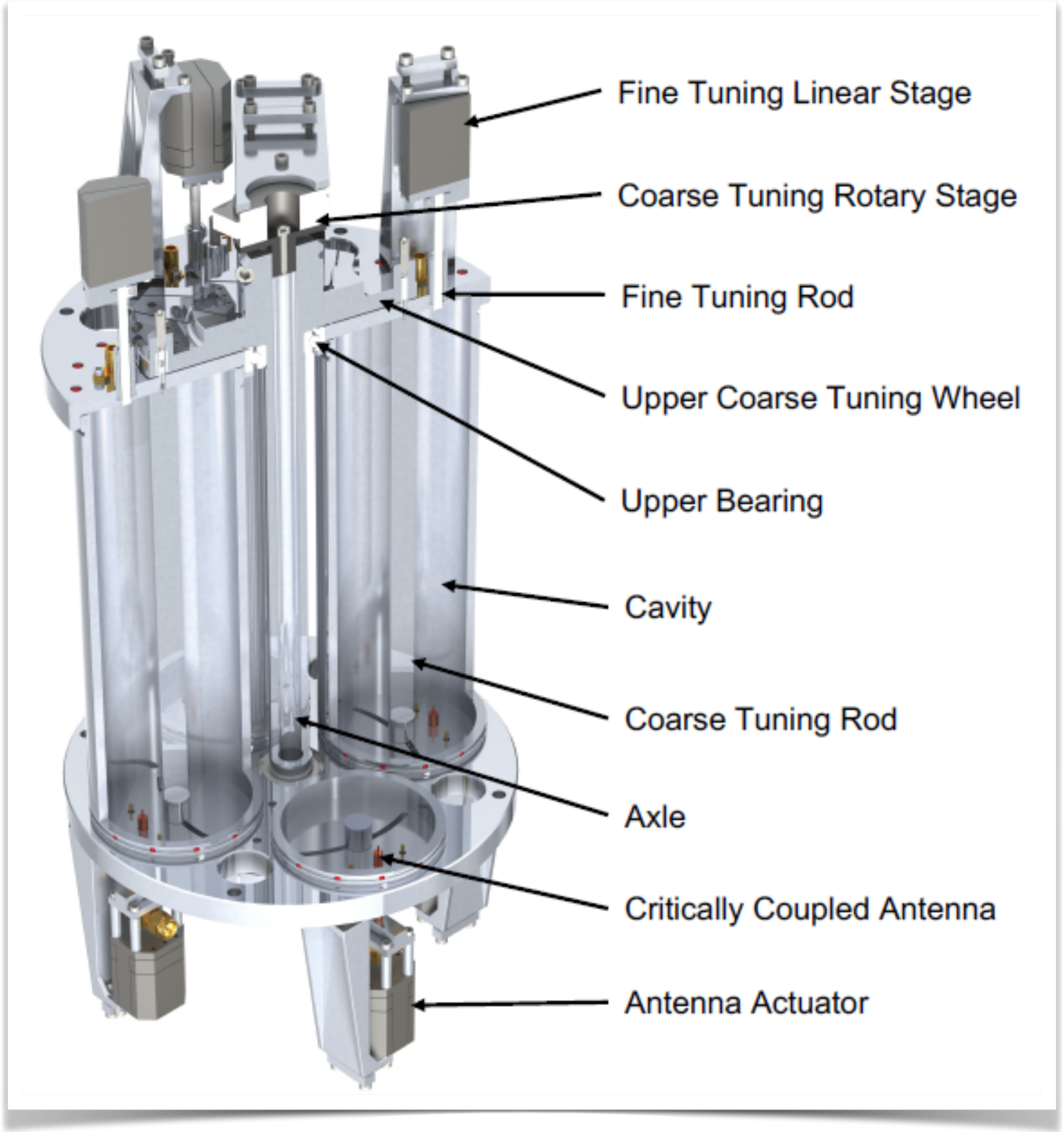
$$T_{\mathrm{sys}} = T_{\mathrm{amp}} + T_{\mathrm{phys}}$$

Maximize	Can't Control	Minimize
• B Field • Volume • Quality Factor • Form Factor	• Frequency • Coupling • Dark Matter Density	• System noise: • Amplifier Noise • Physical Noise

Assume we want to cover a frequency range of 220 MHz in 6 months. What physical temperature is needed?

Axion Search Challenge

High Frequency → Smaller Volume → Slower Scan Rate



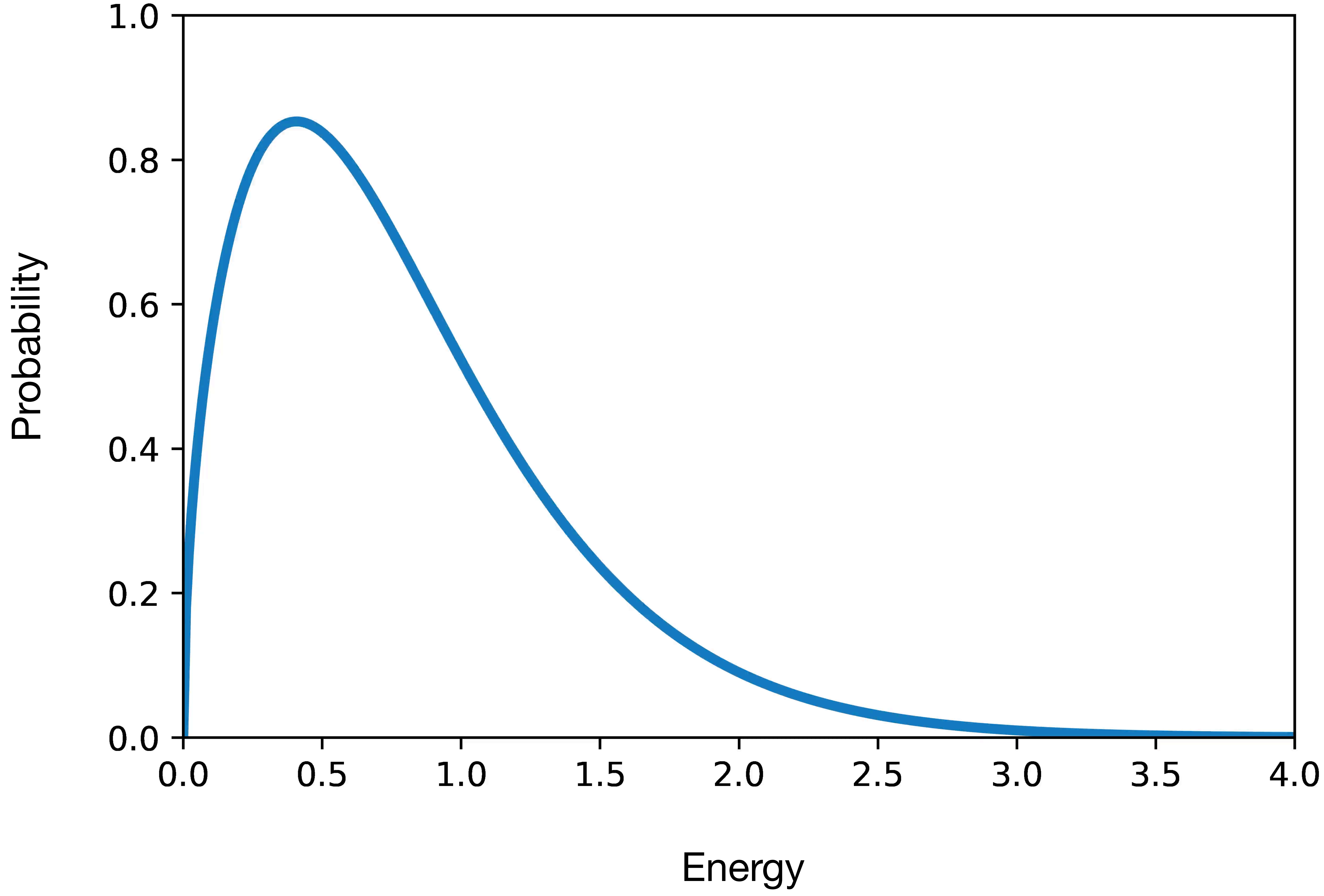
Power-combining multiple cavities is one idea. Can you think of more?

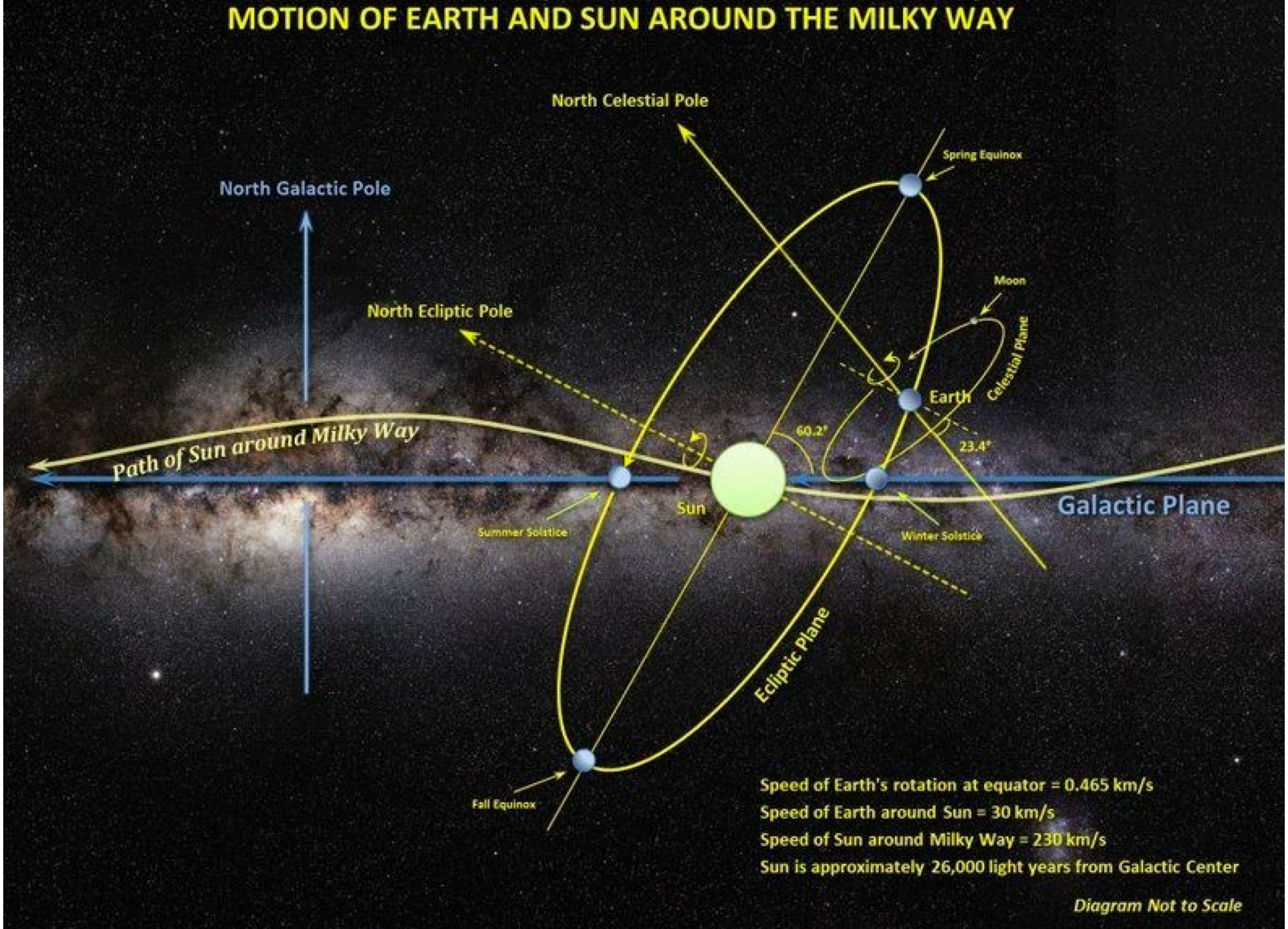
(If you do maybe you will find the axion first and get the Nobel Prize…)

Doppler shift of the axion

Motion of the earth through:
galaxy + orbital + rotational motion = axion doppler shift

- Can be used to further discriminate power excesses.
- High resolution analyses sensitive to the frequency shift.




MOTION OF EARTH AND SUN AROUND THE MILKY WAY
North Celestial Pole
Spring Equinox
North Galactic Pole
Moon
North Ecliptic Pole
Celestial Plane
Earth
Path of Sun around Milky Way
60.2°
23.4°
Sun
Galactic Plane
Summer Solstice
Winter Solstice
Ecliptic Plane
Fall Equinox
Speed of Earth's rotation at equator = 0.465 km/s
Speed of Earth around Sun = 30 km/s
Speed of Sun around Milky Way = 230 km/s
Sun is approximately 26,000 light years from Galactic Center
Diagram Not to Scale

Can compute the frequency shift

$$hf = m_a c^2 + (1/2) m_a \vec{v} \cdot \vec{v}$$

Relative velocity of the axion flow and detector

$$\vec{v} = \vec{v_a} - \vec{v}_{det}$$

Detector velocity

$$\vec{v}_{det} = \vec{v}_{\odot} + \vec{v}e + \vec{v}_U$$

Speed of the detector (Seattle) on the surface of earth

0.3 km/s

Speed of the sun around galaxy

220 km/s

Speed of the earth around the sun

30 km/s

Can compute the frequency shift

$$hf = m_a c^2 + (1/2) m_a \vec{v} \cdot \vec{v}$$

Relative velocity of the axion flow and detector

$$\vec{v} = \vec{v_a} - \vec{v}_{det}$$

Detector velocity

$$\vec{v}_{det} = \vec{v}_{\odot} + \vec{v}e + \vec{v}_U$$

$$hf = m_a c^2 \left(1 + \frac{\vec{v}_a^2 + \vec{v}_{\odot}^2 - 2\vec{v}_a \cdot \vec{v}_{\odot}}{2c^2} + \frac{(\vec{v}_{\odot} - \vec{v}_a) \cdot (\vec{v}_e + \vec{v}_U)}{c^2} \right)$$

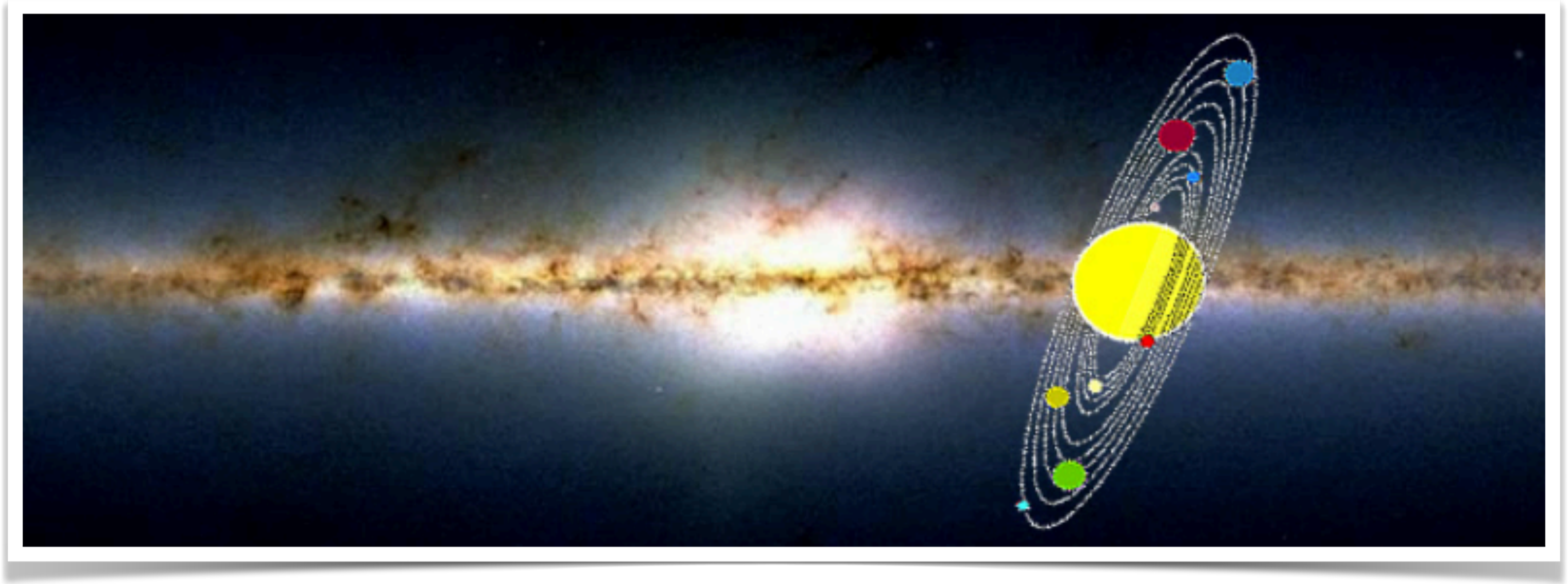
O(10^{-6})

$$\frac{df}{dt} = f_0 \left(\frac{(\vec{v}_{\odot} - \vec{v}_a)}{c^2} \cdot \left(\frac{d\vec{v}_e}{dt} + \frac{d\vec{v_U}}{dt} \right) \right)$$

$$f_0 = \frac{m_a c^2}{h}$$

Calculate the frequency shift of an axion

$$\frac{df}{dt} = f_0 \left(\frac{(\vec{v}_{\odot} - \vec{v}_a)}{c^2} \cdot \left(\frac{d\vec{v}_e}{dt} + \frac{d\vec{v}_U}{dt} \right) \right)$$



Ecliptic angle = 60 degrees
Earth axis angle = 37 degrees

Naive model for dark matter: radial infall (618,0,0) (618 km/s is the escape velocity from our location)

$$\frac{d\vec{v}_e}{dt} \approx \frac{\pm v_e \hat{r} \pm v_e \cos 60^\circ \phi \pm v_e \sin 60^\circ \hat{z}}{3\,\text{months}}$$

V_e = 30 km/s

V_U = 0.3 km/s

$V_{\odot}$ = 220 km/s

V_a = (618,0,0)

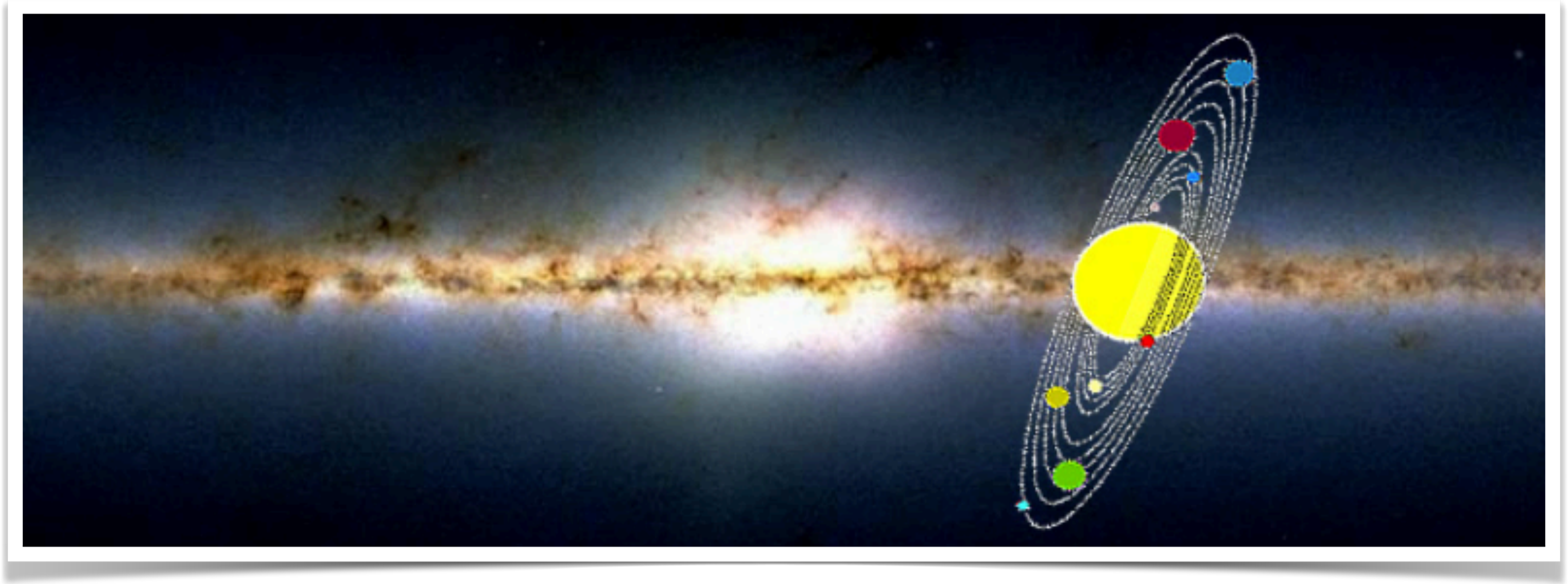
$V_{\odot}$ = (0,220,0)

$$\frac{d\vec{v}_U}{dt} \approx \frac{\pm v_U \hat{r} \pm v_U \cos 37^\circ \phi \pm v_U \sin 37^\circ \hat{z}}{6\,\text{hours}}$$

At a frequency of 692 MHz, how far does the signal shift in 8 minutes?

Calculate the frequency shift of an axion

$$\frac{df}{dt} = f_0 \left(\frac{(\vec{v}_\odot - \vec{v}_a)}{c^2} \cdot \left(\frac{d\vec{v}_e}{dt} + \frac{d\vec{v}_U}{dt} \right) \right)$$



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At a frequency of 692 MHz, how far does the signal shift in 8 minutes?

54.1 mHz

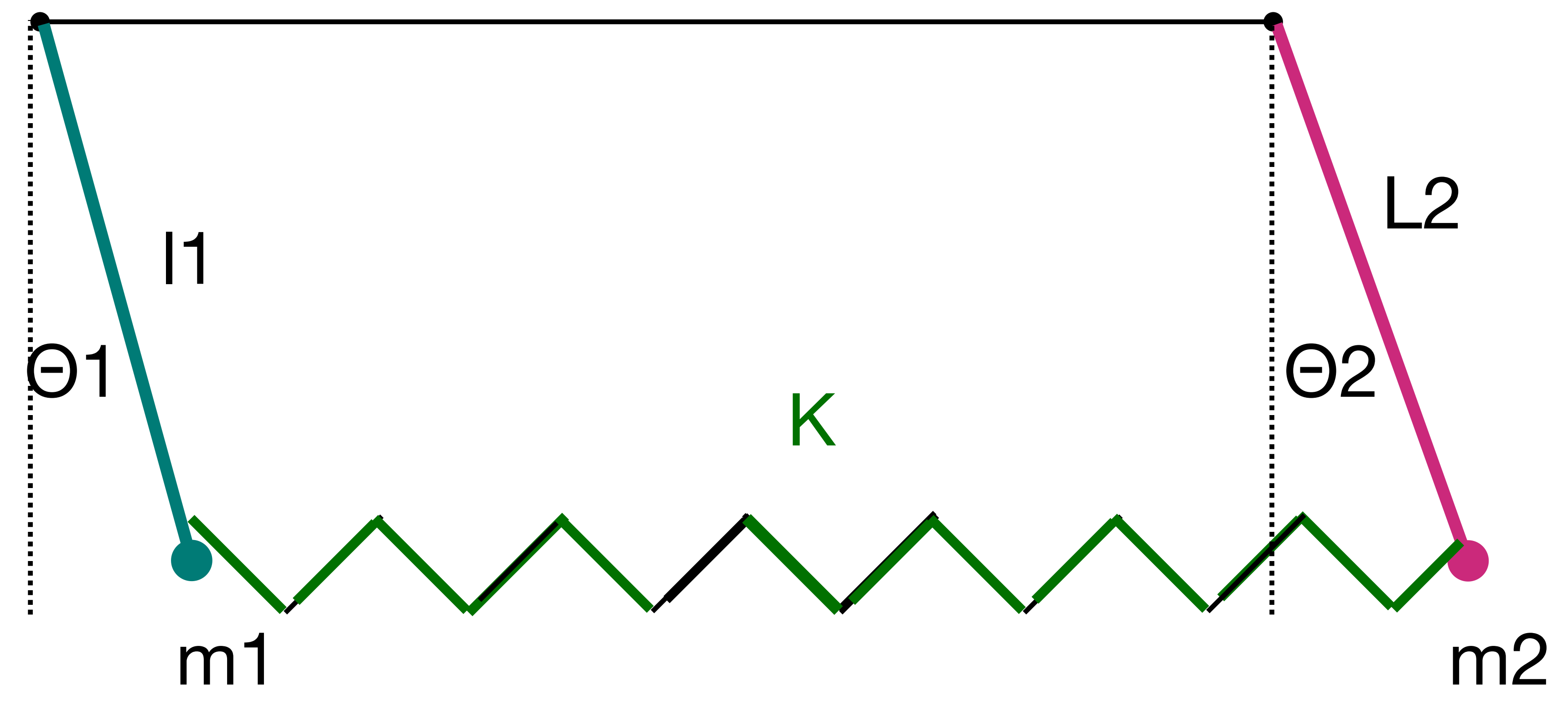
Coupled oscillators

Can describe the total potential energy as

$$V = \frac{m}{2}\left(\frac{g}{L_a}x_a^2 + \frac{g}{L_b}x_b^2 + \frac{k}{m}(x_b - x_a)^2\right)$$

$$V = \frac{m}{2}(x_a\, x_b)\begin{pmatrix} \frac{g}{L_a} + \frac{k}{m} & -\frac{k}{m} \\ -\frac{k}{m} & \frac{g}{L_b} + \frac{k}{m} \end{pmatrix}\begin{pmatrix} x_a \\ x_b \end{pmatrix}$$

Matrix is real and symmetric and so orthogonally diagonalizable



Axion Case

Pendulum 1: Axion
Pendulum 2: Photon
Weak spring: axion-photon coupling

Neutrino Case

Pendulum 1: Electron neutrino
Pendulum 2: Tau neutrino
Weak spring: mixing angle

Coupled oscillators

Can describe the total potential energy as

$$V = \frac{m}{2}\left(\frac{g}{L_a}x_a^2 + \frac{g}{L_b}x_b^2 + \frac{k}{m}(x_b - x_a)^2\right)$$

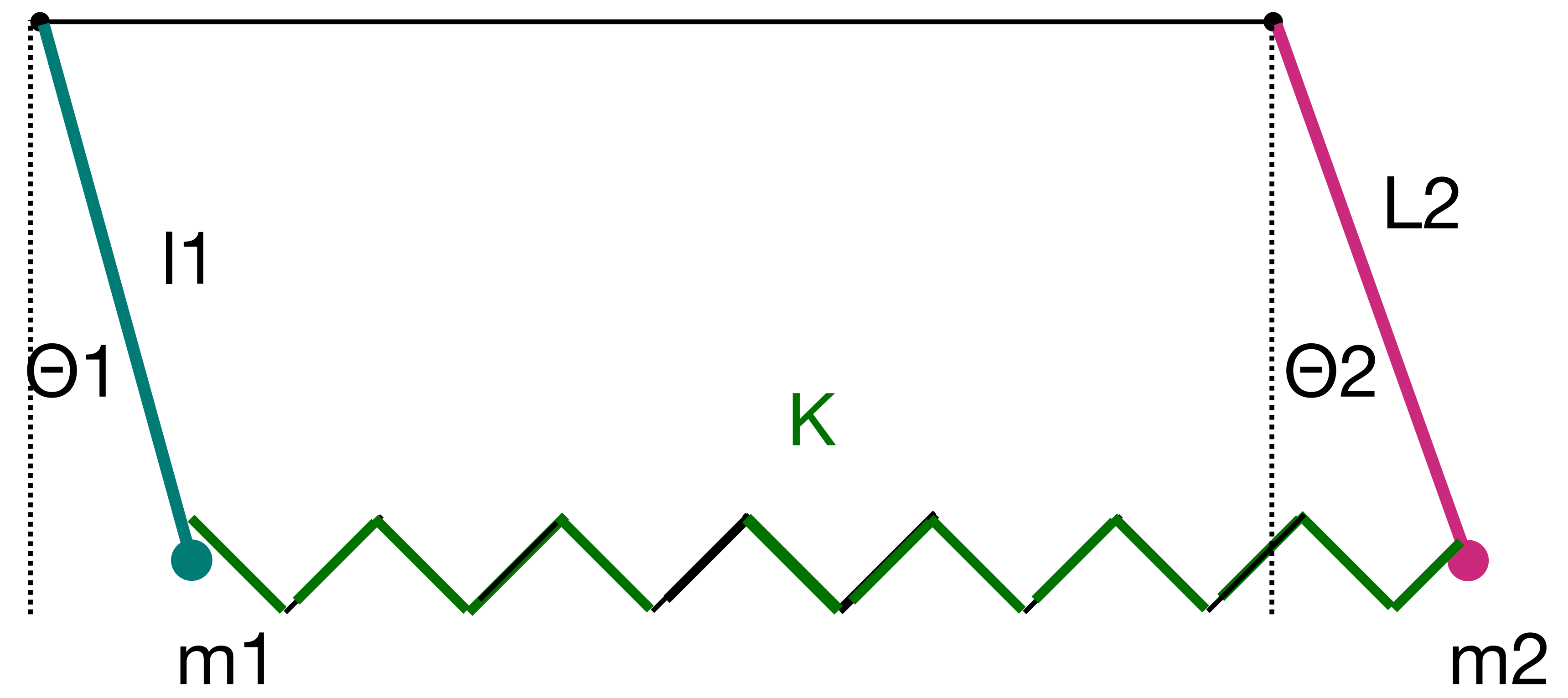
$$V = \frac{m}{2}(x_a\, x_b)\begin{pmatrix} \frac{g}{L_a} + \frac{k}{m} & -\frac{k}{m} \\ -\frac{k}{m} & \frac{g}{L_b} + \frac{k}{m} \end{pmatrix}\begin{pmatrix} x_a \\ x_b \end{pmatrix}$$

Matrix is real and symmetric and so orthogonally diagonalizable

There exists an angle theta such that

$$\begin{pmatrix} x_a \\ x_b \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

We call this the mixing angle.



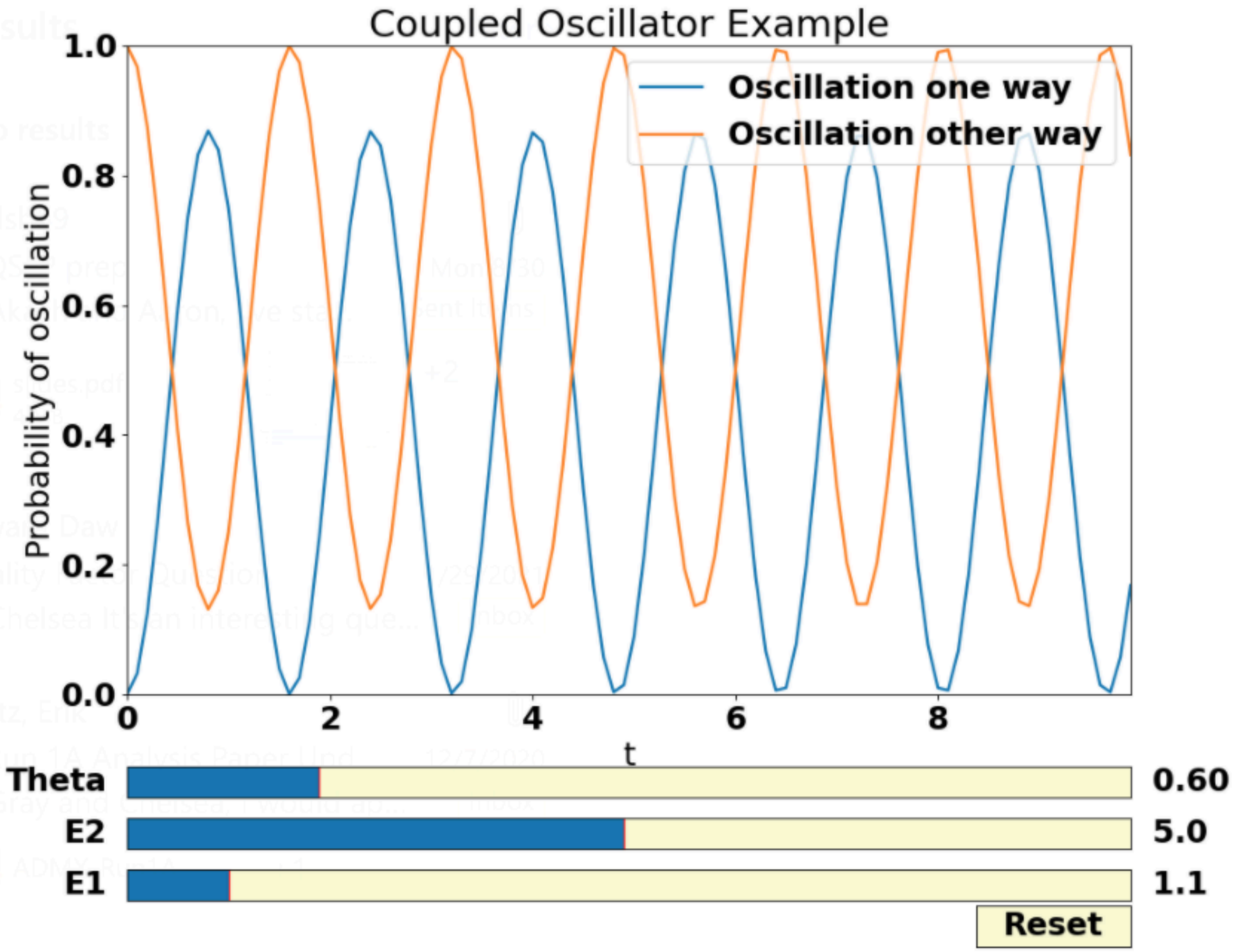
$$V = \frac{m}{2}(x_1\, x_2)\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

x1 and x2 are normal modes which oscillate with frequencies

$$\sqrt{\lambda_1} \qquad \sqrt{\lambda_2}$$

Coupled oscillators

- Axion field and the cavity are like two coupled oscillators
- Generalized coupled oscillator problem:
  - Neutrino oscillation
  - Coupled circuits
  - Two pendula with weak spring



Let's open and run the Jupyter notebook!

Notice that the max transfer of energy is when theta = 45 degrees.
In the axion example, this corresponds to having the cavity on resonance!

Thanks everyone!
Other questions?